Digital Communication

Short notes for B.Tech ECE · GATE EC

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1. Introduction

- **Digital communication** sends information as a sequence of symbols (bits).
- Chain: source → source encoder → channel encoder → modulator → channel → demodulator → channel decoder → source decoder.

➤ Why digital?

1. Regenerative repeaters remove noise at every hop.
2. Error detection and correction is possible.
3. Easy encryption and multiplexing; works with computers.
4. Disadvantage: needs more bandwidth and needs synchronisation.

2. Sampling and Pulse Code Modulation (PCM)

- PCM steps: sampling → quantisation → encoding.
- Sampling rate fs >= 2 fm (Nyquist). Telephone speech: fm ≈ 3.4 kHz, fs = 8 kHz.

➤ Quantisation

- L levels, n bits per sample: L = 2^n. Step size Δ = (Vmax − Vmin) / L = 2 Vm / L.
- Quantisation error lies between −Δ/2 and +Δ/2. Mean square error (noise power) = Δ^2 / 12.

FORMULA SQNR (full-scale sine) = 1.76 + 6.02 n dB. Each extra bit adds about 6 dB.

FORMULA Bit rate Rb = n fs. Minimum transmission bandwidth BT = Rb / 2 = n fs / 2 (Nyquist).

- **Non-uniform quantisation** (companding) gives better SQNR to weak signals. μ-law (μ = 255, North America, Japan) and A-law (A = 87.6, Europe, India).

➤ Solved example (PCM)

- Message fm = 4 kHz, sampled at Nyquist rate, 256 levels.
  - fs = 2 x 4 = 8 kHz, n = log2 256 = 8 bits
  - Bit rate = 8 x 8000 = 64 kbps
  - Minimum bandwidth = 64 / 2 = 32 kHz
  - SQNR ≈ 1.76 + 6.02 x 8 = 49.9 dB

➤ DPCM and delta modulation

- **DPCM**: quantise the difference between the sample and its prediction. Needs fewer bits.
- **Delta modulation (DM)**: 1 bit per sample (step up or step down by Δ). Bit rate = fs.
- **Slope overload**: step cannot follow a fast signal. Avoid if Δ fs >= max slope of m(t).
- **Granular noise**: step too big for a slowly changing signal.
- **Adaptive DM** changes Δ to reduce both problems.

FORMULA No slope overload for $m(t) = Am \sin(2\pi\, fm\, t)$ if $\Delta\, fs >= 2\pi\, fm\, Am$.

3. Baseband Transmission

➤ Line codes

Code	Rule	Note
Unipolar NRZ	1 = +V, 0 = 0	Has DC, poor sync
Polar NRZ	1 = +V, 0 = −V	Less DC if balanced
Unipolar RZ	1 = +V for half bit	Better sync, double bandwidth
Bipolar (AMI)	1 alternates +V, −V; 0 = 0	No DC, single-error detection
Manchester	1 = high-to-low, 0 = low-to-high (one common convention)	Self-clocking, no DC, used in Ethernet

➤ Inter-symbol interference (ISI)

- **ISI**: pulses spread in time and overlap with neighbours because the channel bandwidth is limited.
- **Nyquist criterion** for zero ISI: pulse $p(t)$ is 1 at $t = 0$ and 0 at all other sampling instants nT.
- Ideal Nyquist (sinc) pulse needs bandwidth $Rs / 2$, but it cannot be built and is very timing sensitive.

FORMULA Raised cosine pulse: $BW = (Rs / 2)(1 + \alpha)$, roll-off $0 <= \alpha <= 1$. For binary, $Rs = Rb$.

EXAMPLE $Rb = 10$ kbps binary, $\alpha = 0.5$: $BW = (10 / 2) \times 1.5 = 7.5$ kHz.

- **Eye diagram**: width of opening = time margin, height = noise margin, slope = timing sensitivity.
- **Equaliser** (zero-forcing, adaptive) reduces ISI at the receiver.

4. Optimum Detection

- **Matched filter**: the filter that gives maximum output SNR at the sampling instant. $h(t) = s(T - t)$.

FORMULA Maximum SNR of matched filter $= 2E / N0$ (E = signal energy, $N0/2$ = noise PSD).

- Matched filter output can also be made with a **correlator** (multiply and integrate).
- **Q-function**: $Q(x)$ = probability that a standard Gaussian exceeds x. $Q(0) = 0.5$.

5. Digital Modulation (Passband)

Scheme	What changes	Bandwidth (null-to-null)	Coherent BER
ASK (OOK)	Amplitude (on/off)	2 Rb	Q(sqrt(Eb / N0))
BFSK	Frequency (f1, f2)	About (f1 − f2) + 2 Rb	Q(sqrt(Eb / N0))
BPSK	Phase 0 / π	2 Rb	Q(sqrt(2 Eb / N0))
QPSK	Phase π/4, 3π/4, ... (2 bits)	Rb	Q(sqrt(2 Eb / N0)) (bit error)
DPSK	Phase change vs previous bit	2 Rb	(1/2) e^(−Eb / N0) (non-coherent)
Non-coherent FSK	Frequency, envelope detection	About (f1 − f2) + 2 Rb	(1/2) e^(−Eb / 2N0)

REMEMBER BPSK is best in BER. QPSK has the same BER as BPSK but needs only half the bandwidth.

- **M-ary PSK**: log2 M bits per symbol, bandwidth = 2 Rb / log2 M, but points come closer → more errors.
- **QAM**: changes both amplitude and phase (16-QAM, 64-QAM). High data rate, needs good SNR.
- **MSK**: FSK with minimum spacing Δf = Rb / 2, continuous phase. **GMSK** (Gaussian-filtered MSK) is used in GSM.
- Bandwidth efficiency = Rb / BW (bits/s/Hz).

FORMULA Eb / N0 = (S / N) x (B / Rb). Symbol energy Es = Eb log2 M.

➤ Solved example (BPSK)

- BPSK, received Eb / N0 = 9 (9.54 dB).
  - BER = Q(sqrt(2 x 9)) = Q(sqrt(18)) = Q(4.24)
  - Q(4.24) ≈ 1.1 x 10^-5, so about 1 bit in 90,000 is wrong.
  - For coherent BFSK at the same Eb / N0: Q(sqrt(9)) = Q(3) ≈ 1.35 x 10^-3 (much worse).

6. Information Theory

- **Information** of a symbol with probability p: I = log2(1/p) bits. Rare symbols carry more information.

FORMULA Entropy H = − sum of pi log2 pi (bits/symbol). Maximum = log2 M when all M symbols are equally likely.

FORMULA Information rate R = r H (r = symbols per second).

➤ Solved example (entropy and Huffman)

- Source with 4 symbols: p = 1/2, 1/4, 1/8, 1/8.
  - H = (1/2)(1) + (1/4)(2) + (1/8)(3) + (1/8)(3) = 0.5 + 0.5 + 0.375 + 0.375 = 1.75 bits/symbol
  - Huffman codes: 0, 10, 110, 111. Average length L = 0.5 x 1 + 0.25 x 2 + 0.125 x 3 x 2 = 1.75 bits
  - Efficiency = H / L = 100% (probabilities are powers of 1/2)

➤ Channel capacity

FORMULA Shannon–Hartley: C = B log2(1 + S/N) bits/s.

EXAMPLE B = 3 kHz, S/N = 30 dB = 1000: C = 3000 x log2(1001) = 3000 x 9.97 ≈ 29.9 kbps.

- If R <= C, error-free transmission is possible (with good coding). If R > C, it is not.
- Infinite bandwidth limit: C∞ = 1.44 S / N0. **Shannon limit**: Eb / N0 >= −1.6 dB.
- Binary symmetric channel (error probability p): C = 1 − H(p), where H(p) = −p log2 p − (1 − p) log2(1 − p).
- Mutual information I(X;Y) = H(X) − H(X|Y).

➤ Source coding

- **Huffman**: optimum prefix code (merge the two smallest probabilities again and again).
- **Shannon–Fano**: split symbols into two groups of nearly equal probability.
- Source coding theorem: average code length L >= H.

7. Error Control Coding

- **Hamming distance** = number of positions in which two code words differ. dmin = smallest distance between any two code words.

> **FORMULA** Detect up to s errors: dmin >= s + 1. Correct up to t errors: dmin >= 2t + 1.

➤ Linear block codes (n, k)

- k message bits, n − k parity bits. Code rate = k / n.
- Generator matrix G = [I_k | P]. Parity check matrix H = [P^T | I_(n−k)]. Code word c = m G, and c H^T = 0.
- **Syndrome** s = r H^T. If s = 0, no error detected. Else s points to the error pattern.
- For a linear code, dmin = smallest weight of a non-zero code word.

➤ Hamming codes

- Single error correcting, dmin = 3. Parity bits p must satisfy $2^p >= m + p + 1$.
- (7, 4) code: 4 data bits, 3 parity bits, code rate 4/7.

➤ Cyclic codes and CRC

- Any cyclic shift of a code word is also a code word. Generated by g(x), a factor of x^n + 1.
- **CRC**: append the remainder of m(x) x^(n−k) divided by g(x). Very good at burst error detection.
- **Convolutional codes**: output depends on present and past inputs; decoded by the **Viterbi algorithm**.

Method	Idea	Example use
ARQ	Detect error, ask for retransmission	Data networks
FEC	Detect and correct at receiver	Satellite, mobile

8. Multiplexing and Synchronisation

- **TDM**: T1 = 24 channels, 1.544 Mbps. E1 = 32 slots (30 voice), 2.048 Mbps.
- **Bit sync** (clock recovery), **frame sync** (find frame start), **carrier sync** (for coherent detection).

EXAMPLE E1: 32 slots x 8 bits x 8000 frames/s = 2.048 Mbps.

9. Most Asked Questions (practice)

1. Explain PCM with a block diagram. Find bit rate and SQNR.
2. Derive quantisation noise power $\Delta^2 / 12$ and SQNR = 1.76 + 6.02n dB.
3. What is slope overload in DM? Derive the condition to avoid it.
4. Explain ISI and the Nyquist criterion. What is a raised cosine pulse?
5. Derive the matched filter and its maximum SNR.
6. Compare ASK, FSK, PSK, QPSK and DPSK in bandwidth and BER.
7. Find entropy and construct Huffman code for a given source.
8. State the Shannon–Hartley theorem and find channel capacity.
9. Find code words, syndrome and error correction for a (7, 4) Hamming code.
10. Explain CRC generation with an example.

TIP GATE EC loves entropy, capacity, PCM bit rate, raised cosine bandwidth, matched filter SNR and BPSK/QPSK BER. Learn the BER table and Q-function basics.