Electromagnetic Theory

Short notes for B.Tech ECE · EEE · GATE EC

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1. Vector Analysis

- **Gradient** ∇V: a vector pointing in the direction of the fastest increase of a scalar V.
- **Divergence** ∇·A: net outward flux per unit volume (source strength).
- **Curl** ∇xA: rotation (circulation per unit area) of a vector field.

Theorem	Statement
Divergence (Gauss) theorem	Closed surface integral of A·dS = volume integral of (∇·A) dv
Stokes' theorem	Closed line integral of A·dl = surface integral of (∇xA)·dS

- Identities: ∇·(∇xA) = 0 and ∇x(∇V) = 0.
- **Solenoidal** field: ∇·A = 0. **Irrotational** (conservative) field: ∇xA = 0.

➤ Coordinate systems

System	Coordinates	Differential volume
Cartesian	x, y, z	dx dy dz
Cylindrical	ρ, ϕ, z	ρ dρ dϕ dz
Spherical	r, θ, ϕ	r^2 sin θ dr dθ dϕ

2. Electrostatics

FORMULA Coulomb's law: F = Q1 Q2 / (4π ε0 R^2), ε0 = 8.854 x 10^-12 F/m. E = F / Q.

- Field of a point charge: E = Q / (4π ε0 r^2). Infinite line charge: E = ρL / (2π ε0 ρ). Infinite sheet: E = ρs / (2 ε0).
- **Gauss's law**: total flux out of a closed surface = charge enclosed. Point form: ∇·D = ρv, where D = ε E.

- **Potential**: V = −integral of E·dl, E = −∇V. Point charge: V = Q / (4π ε0 r).
- **Poisson's equation**: ∇^2 V = −ρv / ε. **Laplace's equation**: ∇^2 V = 0 (charge-free region).

FORMULA Energy stored: W = (1/2) integral of ε E^2 dv. Capacitance C = Q / V. Parallel plate C = ε A / d.

➤ Capacitance of common shapes

Shape	Capacitance
Parallel plate	ε A / d
Coaxial (per unit length)	2π ε / ln(b/a)
Isolated sphere	4π ε a
Concentric spheres	4π ε / (1/a − 1/b)

➤ Boundary conditions

Boundary	Tangential	Normal
Dielectric–dielectric	E1t = E2t	D1n − D2n = ρs (= 0 if no surface charge)
Conductor–dielectric	Et = 0	Dn = ρs
Magnetic (two media)	H1t − H2t = K (surface current)	B1n = B2n

- Inside a perfect conductor: E = 0, charge sits on the surface, the surface is an equipotential.
- **Method of images**: a point charge above a grounded plane = charge + an equal and opposite image charge below.

3. Magnetostatics

- **Biot–Savart law**: dH = I dl x aR / (4π R^2).
- **Ampere's circuit law**: closed line integral of H·dl = I enclosed. Point form: ∇xH = J.
- ∇·B = 0 (no magnetic monopoles). B = μ H, μ0 = 4π x 10^-7 H/m.

Source	Magnetic field H
Infinite straight wire	I / (2π ρ)
Centre of a circular loop (radius a)	I / (2a)
Long solenoid (n turns per metre)	n I (inside)
Infinite current sheet K	K / 2

FORMULA Lorentz force: F = Q (E + v x B). Force on a wire: F = I L x B.

- Magnetic vector potential A: B = ∇xA. Inductance L = N Φ / I. Energy W = (1/2) L I^2 = (1/2) integral of μ H^2 dv.

4. Maxwell's Equations

Law	Point (differential) form	Integral form
Gauss (electric)	∇·D = ρv	Closed surface ∮ D·dS = Q enclosed
Gauss (magnetic)	∇·B = 0	∮ B·dS = 0
Faraday	∇xE = −∂B/∂t	∮ E·dl = −d/dt (∫ B·dS)
Ampere–Maxwell	∇xH = J + ∂D/∂t	∮ H·dl = I + ∫ (∂D/∂t)·dS

- **Displacement current density** Jd = ∂D/∂t was added by Maxwell so the equations agree with charge conservation.
- **Continuity equation**: ∇·J = −∂ρv/∂t.
- Time-harmonic (phasor) form: replace ∂/∂t by jω.

REMEMBER Ratio of conduction to displacement current = σ / (ωε). Good conductor: σ/(ωε) >> 1. Good dielectric: << 1.

5. Uniform Plane Waves

- Wave equation in free space: ∇^2 E = μ0 ε0 ∂^2 E / ∂t^2. Speed c = 1 / sqrt(μ0 ε0) = 3 x 10^8 m/s.
- In a plane wave, E, H and the direction of travel are mutually perpendicular (TEM wave).

FORMULA Propagation constant γ = α + jβ = sqrt(jωμ (σ + jωε)). Intrinsic impedance η = sqrt(jωμ / (σ + jωε)).

Medium	α	β	η	Velocity
Free space	0	ω / c	120π ≈ 377 Ω	c
Lossless dielectric	0	ω sqrt(με)	sqrt(μ/ε) = 377 / sqrt(εr) (μr = 1)	c / sqrt(εr)
Good conductor	sqrt(πfμσ)	sqrt(πfμσ)	(1 + j) sqrt(πfμ/σ), angle 45°	ω / β (slow)

FORMULA Skin depth δ = 1 / α = 1 / sqrt(π f μ σ) (good conductor).

➤ Solved example (skin depth)

- Copper, σ = 5.8 x 10^7 S/m, μ = μ0, f = 1 MHz.
  - π f μ σ = 3.1416 x 10^6 x 1.2566 x 10^-6 x 5.8 x 10^7 = 2.29 x 10^8
  - sqrt(2.29 x 10^8) = 1.513 x 10^4 m^-1
  - δ = 1 / 1.513 x 10^4 = 66 μm. So RF current flows only in a thin surface layer.

EXAMPLE In a lossless dielectric with εr = 4, μr = 1: v = c / 2 = 1.5 x 10^8 m/s, η = 377 / 2 ≈ 188.5 Ω.

➤ Power and polarisation

- **Poynting vector** P = E x H (W/m^2). Average power density = (1/2) Re(E x H*) = |E0|^2 / (2η) for a lossless medium.
- Polarisation (direction of E over time): **linear**, **circular** (two equal components, 90° phase difference), **elliptical** (general case).

➤ Reflection at a boundary (normal incidence)

FORMULA Γ = (η2 − η1) / (η2 + η1), τ = 2η2 / (η2 + η1), τ = 1 + Γ.

- Perfect conductor: η2 = 0 → Γ = −1 (total reflection, standing wave).
- **Brewster angle** (parallel polarisation, no reflection): tan θB = sqrt(ε2 / ε1).
- **Critical angle** (total internal reflection): sin θc = sqrt(ε2 / ε1), only when going from a denser to a rarer medium.

6. Transmission Lines

- Distributed parameters per unit length: R, L, G, C.

FORMULA $Z0 = \sqrt{(R + j\omega L) / (G + j\omega C)}$. Lossless line: $Z0 = \sqrt{L/C}$, $\beta = \omega \sqrt{LC}$, $v = 1 / \sqrt{LC}$.

FORMULA Input impedance (lossless): $Zin = Z0 (ZL + j\, Z0 \tan \beta l) / (Z0 + j\, ZL \tan \beta l)$.

FORMULA Reflection coefficient $\Gamma L = (ZL - Z0) / (ZL + Z0)$. VSWR $S = (1 + |\Gamma|) / (1 - |\Gamma|)$.

➤ Special lengths and loads

Case	Zin
Short circuit, length l	$j\, Z0 \tan \beta l$ (inductive for $l < \lambda/4$)
Open circuit, length l	$-j\, Z0 \cot \beta l$ (capacitive for $l < \lambda/4$)
Quarter-wave line ($l = \lambda/4$)	$Z0^2 / ZL$ (impedance inverter)
Half-wave line ($l = \lambda/2$)	ZL (repeats the load)
Matched load $ZL = Z0$	$Z0$ at any length, $\Gamma = 0$, VSWR = 1

- $Z0 \times Z0 = Zsc \times Zoc$ (open and short circuit measurement): $Z0 = \sqrt{Zoc\, Zsc}$.
- Voltage maxima and minima are $\lambda/2$ apart. Distance from a max to the next min = $\lambda/4$.
- **Smith chart**: graphical tool to find Zin, Γ and VSWR; one full turn = $\lambda/2$.
- **Stub matching**: a short-circuited stub placed in parallel cancels the susceptance of the load.

➤ Solved example (mismatch)

- $Z0 = 50\ \Omega$ line ends in $ZL = 100\ \Omega$.
  - $\Gamma = (100 - 50) / (100 + 50) = 1/3$
  - VSWR $= (1 + 1/3) / (1 - 1/3) = (4/3) / (2/3) = 2$
  - Fraction of power reflected $= |\Gamma|^2 = 1/9 \approx 11.1\%$
  - A quarter-wave transformer for matching needs $Z0' = \sqrt{50 \times 100} = 70.7\ \Omega$

7. Waveguides

- A hollow metal pipe guiding EM waves. It cannot carry **TEM** waves (only one conductor). It carries **TE** and **TM** modes.
- Each mode has a **cutoff frequency**. Below cutoff, the wave decays (evanescent).

FORMULA Rectangular guide (a x b): fc(mn) = (c / 2) sqrt((m/a)^2 + (n/b)^2).

FORMULA Guide wavelength λg = λ / sqrt(1 − (fc/f)^2). Phase velocity vp = c / sqrt(1 − (fc/f)^2), group velocity vg = c sqrt(1 − (fc/f)^2), vp vg = c^2.

- **Dominant mode** of a rectangular guide (a > b) = TE10, fc = c / (2a).
- TM modes need m >= 1 and n >= 1 (lowest TM11). TE modes need m or n >= 1.
- Wave impedance: ZTE = η / sqrt(1 − (fc/f)^2), ZTM = η sqrt(1 − (fc/f)^2).

EXAMPLE a = 2.286 cm (X-band guide). TE10 cutoff = 3 x 10^8 / (2 x 0.02286) = 6.56 GHz.

8. Most Asked Questions (practice)

1. State and explain the divergence and Stokes' theorems.
2. Use Gauss's law to find E for a line charge and a sphere of charge.
3. Write boundary conditions for E and H at a dielectric and a conductor surface.
4. Write Maxwell's equations in point and integral form. Explain displacement current.
5. Derive the wave equation and find α, β, η for a lossy medium.
6. Define skin depth and find it for copper at a given frequency.
7. Explain the Poynting theorem and types of polarisation.
8. Derive the input impedance of a transmission line. Find Γ and VSWR.
9. Explain quarter-wave transformer and single stub matching.
10. Find cutoff frequency, λg and wave impedance for a rectangular waveguide.

TIP In GATE EC, transmission line (Γ, VSWR, Zin), waveguide cutoff and plane-wave parameters give easy numericals. Memorise 377 Ω, c and the TE10 result.