Engineering Mechanics

Short notes for all 1st-year B.Tech branches · GATE ME

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1. Basics and Force Systems

- **Mechanics** = study of bodies at rest (**statics**) or in motion (**dynamics**).
- **Rigid body**: distance between any two points never changes. We assume this in statics.
- **Force** = push or pull. It has magnitude, direction, point of application and line of action. Unit: newton (N).
- **Scalar** has only size (mass, time). **Vector** has size and direction (force, velocity).

➤ Types of force systems

System	Lines of action	Example
Collinear	Same line	Tug of war rope
Concurrent	Meet at one point	Strings tied to a ring
Parallel	Parallel lines	Loads on a beam
Coplanar non-concurrent	Same plane, do not meet	Ladder against a wall
Non-coplanar	Different planes	Tripod legs

➤ Important laws

- **Parallelogram law**: two forces at a point act like the diagonal of the parallelogram they make.
- **Triangle law**: if two forces are drawn tip to tail, the closing side (reverse) is the resultant.
- **Principle of transmissibility**: a force can be moved along its own line of action without changing its effect on a rigid body.
- **Varignon's theorem**: moment of the resultant about a point = sum of moments of all the forces about that point.

FORMULA Resultant of two forces P and Q at angle θ: R = sqrt(P^2 + Q^2 + 2PQ cosθ), tanα = Q sinθ / (P + Q cosθ).

FORMULA Resultant of many coplanar forces: $R = \sqrt{(\Sigma Fx)^2 + (\Sigma Fy)^2}$, $\tan\alpha = \Sigma Fy / \Sigma Fx$.

➤ Moment and couple

- **Moment** = force x perpendicular distance from the point. Unit: N·m.
- **Couple** = two equal, opposite, parallel forces not on the same line. Moment of couple = F x d, the same about every point.
- A couple cannot be balanced by a single force, only by another equal and opposite couple.
- A force at a point = same force at another point + a couple (force–couple system).

2. Equilibrium

- A body is in **equilibrium** when the resultant force and resultant moment are both zero.

FORMULA Conditions of equilibrium (2D): $\Sigma Fx = 0$, $\Sigma Fy = 0$, $\Sigma M = 0$.

- **Free body diagram (FBD)**: draw the body alone and show every external force on it (weight, reactions, friction, applied loads). Always start a problem with the FBD.
- **Two-force member**: forces must be equal, opposite and collinear. **Three-force member**: the three forces must be concurrent (or parallel).

➤ Lami's theorem

- If three concurrent forces keep a body in equilibrium, each force is proportional to the sine of the angle between the other two.

FORMULA $P / \sin\alpha = Q / \sin\beta = R / \sin\gamma$ (α is the angle between Q and R, and so on).

➤ Types of supports

Support	Reactions	Unknowns
Roller	One force, normal to surface	1
Hinge / pin	Two force components	2
Fixed	Two forces + one moment	3

EXAMPLE A 100 N weight hangs from two strings at 30° and 60° to the horizontal. Find the tensions.

- Angle between T1 and T2 = 180 − 30 − 60 = 90°. Angle between T1 and W = 120°. Angle between T2 and W = 150°.
- Lami: T1 / sin150 = T2 / sin120 = 100 / sin90.
- T1 = 100 × 0.5 = 50 N. T2 = 100 × 0.866 = 86.6 N.
- Check: horizontal 50 cos30 = 43.3 and 86.6 cos60 = 43.3. Vertical 25 + 75 = 100 N. Correct.

3. Friction

- **Friction** = force that opposes relative motion (or the tendency of motion) between surfaces in contact.
- **Static friction** acts before motion. **Limiting friction** = maximum static friction, just before sliding. **Kinetic (dynamic) friction** acts during sliding and is a little less than limiting friction.

➤ Laws of dry (Coulomb) friction

1. Limiting friction $F = \mu N$ (N = normal reaction).
2. It does not depend on the area of contact.
3. It depends on the nature of the surfaces.
4. Kinetic friction is nearly independent of speed (for moderate speeds).

FORMULA $\mu = F / N = \tan\phi$, where ϕ = angle of friction. Angle of repose $\alpha = \phi$.

- **Angle of repose**: the largest slope angle at which a body stays at rest on an inclined plane. A block slides down when the slope angle $\theta > \phi$.
- **Cone of friction**: cone of half angle ϕ around the normal. The resultant reaction must lie inside it for no slip.

➤ Body on an inclined plane (angle θ)

- Force along the plane to just push up: $P = W(\sin\theta + \mu\cos\theta)$.
- Force along the plane to just stop sliding down: $P = W(\sin\theta - \mu\cos\theta)$.

EXAMPLE A 10 kg block on a 30° incline, $\mu = 0.2$. Will it slide? Find its acceleration.

- $\tan 30 = 0.577 > 0.2$, so it slides.
- $a = g(\sin\theta - \mu\cos\theta) = 9.81(0.5 - 0.2 \times 0.866) = 9.81 \times 0.3268 = 3.21\ m/s^2$.

➤ Other friction topics

- **Ladder friction**: take moments about the foot; friction acts at the floor (and wall if rough), opposite to the slipping direction.
- **Wedge friction**: draw FBD of the wedge and the block separately; friction acts along each contact surface.
- **Belt friction**: tension ratio grows exponentially with the angle of wrap.

FORMULA Belt / rope: $T1 / T2 = e^{(\mu\theta)}$, θ in radians ($T1$ = tight side).

4. Trusses

- **Truss** = frame of straight members joined at ends by pins, loaded only at joints. Every member is a two-force member (pure tension or compression).
- **Perfect truss**: just enough members to be stable. Fewer = deficient, more = redundant.

FORMULA Perfect (plane) truss: $m = 2j - 3$ (m = members, j = joints).

➤ Methods of analysis

Method	How	Best for
Method of joints	$\Sigma Fx = 0$, $\Sigma Fy = 0$ at each joint	All member forces
Method of sections	Cut through max 3 unknown members, use $\Sigma M = 0$	Force in a few chosen members
Graphical	Force polygon (Maxwell diagram)	Quick drawings

- **Zero-force members**: at a joint with two non-collinear members and no load, both are zero. At a joint with three members, two collinear and no load, the third is zero.

TIP In method of sections, take moments about the point where two of the unknown forces meet; the third one comes out directly.

5. Centroid and Centre of Gravity

- **Centre of gravity (CG)**: point where the whole weight acts. **Centroid**: geometric centre of an area. They coincide for a uniform body.

FORMULA $\bar{x} = \Sigma(A\,x) / \Sigma A$, $\bar{y} = \Sigma(A\,y) / \Sigma A$. For a hole, take its area as negative.

Shape	Centroid position
Rectangle	b/2, d/2 from a corner
Triangle	h/3 from the base
Semicircle	4r / (3π) from the diameter
Quarter circle	4r / (3π) from each straight edge
Solid hemisphere	3r / 8 from the flat face
Solid cone	h / 4 from the base

- **Pappus–Guldinus theorems**: surface area = length of curve x distance moved by its centroid; volume = area x distance moved by its centroid.

6. Moment of Inertia (Area)

- **Area moment of inertia** $I = \Sigma\, A\, y^2$ (second moment of area). Used in bending of beams.
- **Radius of gyration** $k = \sqrt{I / A}$.

FORMULA Parallel axis theorem: $I = I_g + A\,h^2$. Perpendicular axis theorem: $I_z = I_x + I_y$ (for plane areas).

Shape	I about centroidal axis	I about base / other
Rectangle b x d	$bd^3 / 12$	$bd^3 / 3$ about base
Triangle	$bh^3 / 36$	$bh^3 / 12$ about base
Circle (dia d)	$\pi d^4 / 64$	Polar $J = \pi d^4 / 32$
Semicircle	$0.11\,r^4$	$\pi r^4 / 8$ about diameter

Mass moment of inertia

Body	Axis	I
Thin rod, length L	Through centre, normal to rod	ML^2 / 12
Thin rod	Through one end	ML^2 / 3
Solid disc / cylinder	Own axis	MR^2 / 2
Thin ring	Own axis	MR^2
Solid sphere	Diameter	2MR^2 / 5

7. Virtual Work

- **Principle of virtual work**: for a body in equilibrium, the total work done by all forces in any small imaginary (virtual) displacement is zero.
- Useful for ladders, linkages and beams: reactions at frictionless supports do no work, so they drop out.

8. Kinematics of Particles

- **Kinematics** = motion without looking at forces. **Kinetics** = motion with forces.

FORMULA Uniform acceleration: $v = u + at$, $s = ut + (1/2)at^2$, $v^2 = u^2 + 2as$.

Projectile motion (no air resistance)

FORMULA Time of flight $T = 2u\sin\theta / g$, max height $H = u^2 \sin^2\theta / (2g)$, range $R = u^2 \sin 2\theta / g$.

- Range is maximum at $\theta = 45°$. Two angles θ and $(90° - \theta)$ give the same range.

EXAMPLE A ball is thrown at 20 m/s at 30° to the ground. Find time of flight, height and range.

- $T = 2 \times 20 \times 0.5 / 9.81 =$ **2.04 s**.
- $H = 400 \times 0.25 / (2 \times 9.81) =$ **5.10 m**.
- $R = 400 \times \sin 60 / 9.81 = 346.4 / 9.81 =$ **35.3 m**.

Curvilinear and rotational motion

- Normal (centripetal) acceleration = v^2 / r; tangential acceleration = dv/dt.

- Rotation: ω = ω0 + αt, θ = ω0 t + (1/2)αt^2, ω^2 = ω0^2 + 2αθ. Linear speed v = rω.
- **Relative velocity**: velocity of A relative to B = vA − vB (vector subtraction).
- **Instantaneous centre (IC)**: a point about which a body in plane motion is purely rotating at that instant; v = ω x distance from IC.

9. Kinetics of Particles and Rigid Bodies

- **Newton's 2nd law**: F = ma. For rotation: T = Iα.
- **D'Alembert's principle**: add an inertia force (−ma) and treat the problem as statics.

➤ Energy and momentum methods

Method	Statement	Use when
Work–energy	Work done = change in KE	Forces and distance known
Impulse–momentum	F x t = m(v − u)	Forces and time known
Conservation of momentum	Total momentum stays same if no external force	Collisions, guns
Conservation of energy	KE + PE = constant (no friction)	Pendulum, falling bodies

FORMULA KE = (1/2)mv^2 (translation) + (1/2)Iω^2 (rotation). Rolling without slip: v = rω.

➤ Collision (impact)

- **Coefficient of restitution** e = (velocity of separation) / (velocity of approach) = (v2 − v1) / (u1 − u2).
- e = 1 perfectly elastic (KE conserved), e = 0 perfectly plastic (bodies stick), 0 < e < 1 for real bodies.

REMEMBER In every collision, momentum is conserved. KE is conserved only when e = 1.

10. Most Asked Questions (practice)

1. State and prove Lami's theorem. Solve a problem with a weight held by two strings.
2. State Varignon's theorem and the principle of transmissibility.
3. Find the resultant of a coplanar non-concurrent force system and its position.
4. Define angle of friction and angle of repose. Show they are equal.
5. Solve a ladder friction problem (rough floor, smooth wall).
6. Find forces in a truss by the method of joints and by the method of sections.
7. Find the centroid of a T-section, L-section or an area with a hole.
8. Find the moment of inertia of an I-section about its centroidal axes.
9. Derive the range and max height of a projectile.
10. Explain coefficient of restitution. Solve a direct central impact problem.

TIP For exams and GATE, practise FBDs, friction on inclines, truss member forces, centroid/MOI of built-up sections and projectile numericals. Always check units and signs at the end.