Fluid Mechanics and Hydraulics

Short notes for B.Tech Civil · GATE CE

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I. Fluid Properties

- **Fluid** = a substance that deforms continuously under any shear stress, however small.
- **Density** ρ = mass/volume (water 1000 kg/m³). **Specific weight** γ = ρg (water 9.81 kN/m³).
- **Specific gravity** = ρ / ρwater (mercury 13.6).

➤ Viscosity

- **Newton's law of viscosity**: $\tau = \mu \, du/dy$. μ = dynamic viscosity (Pa·s or N·s/m²).
- **Kinematic viscosity** ν = μ / ρ (m²/s; 1 stoke = 10^-4 m²/s).
- Viscosity of liquids falls with temperature; viscosity of gases rises with temperature.

Fluid type	Behaviour
Newtonian	τ linear with du/dy (water, air)
Dilatant	Viscosity rises with shear rate (quicksand, starch paste)
Pseudo-plastic	Viscosity falls with shear rate (blood, paint)
Bingham plastic	Needs yield stress before flow (toothpaste, sewage sludge)
Ideal fluid	No viscosity, incompressible (imaginary)

➤ Surface tension and capillarity

- Pressure inside a droplet p = 4σ/d, soap bubble p = 8σ/d, liquid jet p = 2σ/d.

FORMULA Capillary rise $h = 4\sigma \cos\theta / (\rho g d)$. Water rises (θ = 0), mercury falls (θ about 130°).

- **Bulk modulus** K = −dp / (dV/V). Compressibility = 1/K.

2. Fluid Statics

- **Pascal's law**: pressure at a point is the same in all directions (fluid at rest).
- Hydrostatic law: $dp/dz = -\rho g$, so $p = \rho g h$.
- Absolute pressure = atmospheric + gauge pressure. Vacuum = negative gauge.
- Measuring devices: piezometer (small positive pressure), U-tube manometer, differential manometer, inverted U-tube (light liquid), Bourdon gauge.

➤ Force on submerged plane surface

FORMULA Total force $F = \rho g A \bar{x}$. Centre of pressure depth $h^* = \bar{x} + I_G / (A \bar{x})$ (vertical surface). Inclined: $h^* = \bar{x} + I_G \sin^2\theta / (A \bar{x})$.

- Centre of pressure always lies below the centroid. For a vertical rectangle from free surface, $h^* = 2H/3$.
- Curved surface: horizontal force = force on vertical projection; vertical force = weight of fluid above the surface.

➤ Buoyancy and stability

- **Archimedes**: buoyant force = weight of fluid displaced, acting at the centre of buoyancy B.
- **Metacentric height** $GM = BM - BG$, where $BM = I / V$ (I = MOI of waterplane about the tilt axis, V = displaced volume).

Condition	Floating body
$GM > 0$ (M above G)	Stable
$GM = 0$	Neutral
$GM < 0$	Unstable

- Submerged body: stable only when B is above G.

3. Fluid Kinematics

Flow	Meaning
Steady / unsteady	Properties at a point do not / do change with time
Uniform / non-uniform	Velocity does not / does change along the flow
Laminar / turbulent	Smooth layers / random mixing
Rotational / irrotational	Fluid particles rotate / do not rotate

- **Streamline**: tangent gives velocity direction. **Pathline**: path of one particle. **Streakline**: locus of particles passing a fixed point. In steady flow all three coincide.
- **Continuity equation**: A1 V1 = A2 V2 (incompressible). In 2D: $\partial u/\partial x + \partial v/\partial y = 0$.
- **Stream function** ψ: $u = -\partial\psi/\partial y$, $v = \partial\psi/\partial x$ (one sign convention). ψ exists for any 2D incompressible flow.
- **Velocity potential** ϕ: $u = -\partial\phi/\partial x$, $v = -\partial\phi/\partial y$. ϕ exists only for irrotational flow.
- Lines of constant ψ and constant ϕ cut at right angles (flow net).
- Acceleration = local $(\partial V/\partial t)$ + convective $(u\,\partial V/\partial x + \ldots)$.

4. Fluid Dynamics

- **Euler's equation** along a streamline: $dp/\rho + g\,dz + V\,dV = 0$.
- **Bernoulli's equation** (steady, ideal, incompressible, along a streamline):

FORMULA $p/(\rho g) + V^2/(2g) + z$ = constant (pressure head + velocity head + datum head).

- **Hydraulic gradient line (HGL)** = $p/\rho g + z$. **Energy gradient line (EGL)** = HGL + $V^2/2g$. EGL is always above HGL by $V^2/2g$.

➤ Flow measuring devices

- **Venturimeter**: Cd about 0.95 to 0.98. **Orifice meter**: Cd about 0.6 to 0.65 (more loss, cheaper).

FORMULA Venturimeter $Q = Cd\ a1\ a2\ \sqrt{2gh} / \sqrt{a1^2 - a2^2}$. Differential head with mercury manometer: $h = x\,(Sm/S - 1)$.

- **Pitot tube** measures velocity: $V = Cv\ \sqrt{2gh}$.

- **Notches**: rectangular Q = (2/3) Cd L sqrt(2g) H^(3/2); triangular (V) Q = (8/15) Cd tan(θ/2) sqrt(2g) H^(5/2).

EXAMPLE Venturimeter 300 mm x 150 mm, differential head 2 m of water, Cd = 0.98. a1 = 0.07069 m², a2 = 0.01767 m². sqrt(2gh) = sqrt(39.24) = 6.264 m/s. Qth = a2 x 6.264 / sqrt(1 - (1/4)²) = 0.11069/0.9682 = 0.1143 m³/s. Q = 0.98 x 0.1143 = 0.112 m³/s.

- **Momentum equation**: ΣF = ρQ(V2 − V1). Used for forces on pipe bends and jets.
- Force of a jet on a fixed flat plate (normal) = ρ a V². On a moving plate = ρ a (V − u)².

5. Flow Through Pipes

- **Reynolds number** Re = ρVD/μ = VD/ν. Pipe flow: laminar Re < 2000, transitional 2000 to 4000, turbulent > 4000.

➤ Laminar flow (Hagen–Poiseuille)

- Velocity profile is parabolic. Vmax = 2 × Vavg. Shear stress varies linearly, zero at centre.

FORMULA Head loss hf = 32 μ V L / (ρ g D²). Darcy friction factor f = 64 / Re (laminar).

➤ Turbulent flow and losses

FORMULA Darcy–Weisbach hf = f L V² / (2 g D). (Some books use 4f; then f = 4f.)

- f depends on Re and relative roughness (Moody chart). Smooth turbulent: Blasius f = 0.316 / Re^0.25.

Minor loss	Head loss
Sudden expansion	$(V1 - V2)^2 / 2g$
Sudden contraction	$(1/Cc - 1)^2 V2^2/2g$ (about $0.5 V2^2/2g$)
Pipe entrance	$0.5 V^2/2g$
Pipe exit	$V^2/2g$
Bends, valves	$K V^2/2g$

- **Pipes in series**: same Q, losses add. **Pipes in parallel**: same head loss, Q adds.
- **Equivalent pipe** (Dupuit): L/D^5 = L1/D1^5 + L2/D2^5 + ...
- **Max power transmission** through a pipe when hf = H/3. Efficiency then = 66.7%.
- **Water hammer**: pressure rise when a valve is closed suddenly; rigid pipe $p = \rho C V$, $C = \sqrt{K/\rho}$.

EXAMPLE Pipe D = 0.3 m, L = 1000 m, V = 2 m/s, f = 0.02. hf = 0.02 x 1000 x 2^2 / (2 x 9.81 x 0.3) = 80 / 5.886 = 13.59 m.

➤ Boundary layer

- Thickness where velocity reaches 99% of free stream. Laminar: $\delta = 5x / \sqrt{Re_x}$ (Blasius).
- Displacement thickness δ^* > momentum thickness θ. Shape factor $H = \delta^*/\theta$ (2.59 for laminar flat plate).
- Separation happens under an adverse pressure gradient ($dp/dx > 0$).

6. Open Channel Flow

- Free surface at atmospheric pressure. Driving force = gravity (bed slope).
- **Hydraulic radius** R = A / P. **Hydraulic depth** D = A / T (T = top width).

FORMULA Chezy $V = C \sqrt{R S}$. Manning $V = (1/n) R^{(2/3)} S^{(1/2)}$. Relation: $C = R^{(1/6)} / n$.

➤ Most economical sections

Section	Condition
Rectangular	b = 2y, R = y/2
Trapezoidal	Half of a regular hexagon, side slope 60°, R = y/2
Triangular	Side slope 45° (vertex angle 90°)

Specific energy and critical flow

- **Specific energy** $E = y + V^2/(2g)$ (energy above the channel bed).
- **Froude number** $Fr = V / \sqrt{g D}$. $Fr < 1$ subcritical, $Fr = 1$ critical, $Fr > 1$ supercritical.

FORMULA Rectangular channel: critical depth $y_c = (q^2/g)^{(1/3)}$, $E_{min} = 1.5\, y_c$, $V_c = \sqrt{g\, y_c}$.

EXAMPLE $q = 2\ m^2/s$ per metre width. $y_c = (2^2/9.81)^{(1/3)} = (0.4077)^{(1/3)} = 0.741$ m. $E_{min} = 1.5 \times 0.741 = 1.112$ m.

Hydraulic jump

- Sudden rise of water from supercritical to subcritical flow. Used to dissipate energy below spillways.

FORMULA Rectangular channel: $y_2/y_1 = 0.5\,[-1 + \sqrt{1 + 8\,Fr_1^2}]$. Energy loss $\Delta E = (y_2 - y_1)^3 / (4\, y_1\, y_2)$.

EXAMPLE $y_1 = 0.5$ m, $V_1 = 6$ m/s. $Fr_1 = 6/\sqrt{9.81 \times 0.5} = 2.709$. $y_2 = 0.25 \times [-1 + \sqrt{1 + 58.71}] = 0.25 \times 6.727 = 1.682$ m. Loss $= (1.182)^3 / (4 \times 0.5 \times 1.682) = 1.651/3.364 = 0.491$ m.

- **Gradually varied flow (GVF)**: $dy/dx = S_0\,(1 - (y_n/y)^{(10/3)}) / (1 - (y_c/y)^3)$ form. Profiles: M (mild), S (steep), C (critical), H (horizontal), A (adverse). M1 = backwater curve behind a dam.

7. Dimensional Analysis and Model Studies

- **Buckingham π theorem**: n variables, m fundamental dimensions → $(n - m)$ dimensionless π terms.
- Similarity: geometric (shape), kinematic (velocity), dynamic (force ratios).

Model law	Used for	Velocity ratio Vr
Froude	Open channels, spillways, ships (gravity)	sqrt(Lr)
Reynolds	Pipe flow, submarines (viscosity)	vr / Lr
Weber	Capillary, small waves (surface tension)	—
Mach	High-speed flow (compressibility)	—

FORMULA Froude model: Vr = Lr^0.5, Tr = Lr^0.5, Qr = Lr^2.5, Fr (force) = Lr³, Pr (power) = Lr^3.5.

8. Hydraulic Machines

- **Turbines** convert hydraulic energy to mechanical energy. **Pumps** do the reverse.

Turbine	Type	Head	Specific speed
Pelton wheel	Impulse, tangential	High	Low (about 10 to 35)
Francis	Reaction, mixed flow	Medium	Medium (about 60 to 300)
Kaplan / propeller	Reaction, axial	Low	High (about 300 to 1000)

FORMULA Turbine specific speed Ns = N sqrt(P) / H^(5/4). Pump specific speed Ns = N sqrt(Q) / H^(3/4).

- **Draft tube** (reaction turbines) recovers kinetic energy and lets the turbine sit above tail water.
- **Cavitation**: vapour bubbles form when pressure drops below vapour pressure and collapse, damaging blades. Checked by Thoma's cavitation factor.
- Centrifugal pumps need **priming**. Pumps in series raise head; in parallel raise discharge.

9. Most Asked Questions (practice)

1. State Newton's law of viscosity. Classify fluids with a rheology diagram.
2. Find total pressure and centre of pressure on a vertical / inclined plate.
3. Derive the metacentric height of a floating body. State stability conditions.
4. Derive Bernoulli's equation from Euler's equation. List its assumptions.
5. Derive the discharge formula for a venturimeter.
6. Derive the Hagen–Poiseuille equation for laminar flow in pipes.

7. Find the head loss for pipes in series and parallel.
8. Derive conditions for the most economical rectangular and trapezoidal channel.
9. Derive the hydraulic jump depth ratio and energy loss.
10. Use Buckingham π theorem to find the drag force expression; explain Froude model law.

TIP In GATE CE, open channel (critical depth, jump, Manning), pipe losses and Froude model ratios are asked every year. Keep units consistent and practise quick calculator steps.