Fluid Mechanics and Machinery

Short notes for B.Tech Mechanical · GATE ME

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I. Fluid Properties

- **Fluid** = a substance that deforms continuously under any shear stress, however small (liquids and gases).
- **Density** ρ = mass / volume (water 1000 kg/m^3). **Specific weight** w = ρg. **Specific gravity** S = ρ / ρwater.
- **Viscosity**: resistance to flow (internal friction).

FORMULA Newton's law of viscosity: τ = μ (du/dy). Kinematic viscosity ν = μ / ρ (m^2/s; 1 stoke = 10^-4 m^2/s).

- Viscosity of liquids falls with temperature (weaker cohesion). Viscosity of gases rises with temperature (more molecular momentum exchange).

➤ Types of fluids

Type	Behaviour	Example
Ideal	No viscosity, incompressible	Imaginary
Newtonian	τ proportional to du/dy	Water, air, oil
Non-Newtonian	τ not linear with du/dy	Blood, paint, toothpaste
Bingham plastic	Flows only after a yield stress	Toothpaste, drilling mud

- **Surface tension** σ (N/m): due to cohesion at a free surface. Pressure inside a droplet Δp = 4σ/d; soap bubble Δp = 8σ/d; liquid jet Δp = 2σ/d.

FORMULA Capillary rise h = 4σ cosθ / (ρ g d). Water rises (θ ≈ 0); mercury falls (θ > 90°).

- **Bulk modulus** K = −dp / (dV/V). Compressibility = 1/K.

- **Vapour pressure**: when local pressure falls below it, the liquid boils and **cavitation** happens.

2. Fluid Statics

- **Pascal's law**: at a point in a fluid at rest, pressure is the same in all directions.
- **Hydrostatic law**: $dp/dz = -\rho g$, so $p = \rho g h$ below a free surface.
- Absolute pressure = gauge pressure + atmospheric pressure. Vacuum = negative gauge.
- Devices: piezometer, U-tube manometer, differential manometer, inverted manometer, Bourdon gauge.

➤ Hydrostatic force on submerged plane surfaces

FORMULA Total force $F = \rho g A \bar{h}$. Centre of pressure $h^* = \bar{h} + I_G / (A \bar{h})$ (vertical surface). For inclined: $h^* = \bar{h} + I_G \sin^2\theta / (A \bar{h})$.

- Centre of pressure is always below the centroid ($h^* > \bar{h}$).

EXAMPLE Vertical gate 2 m wide, 3 m deep, top edge at the water surface. Find force and its location.

- $A = 6\ m^2$, $\bar{h} = 1.5\ m$. $F = 1000 \times 9.81 \times 6 \times 1.5 =$ 88.3 kN.
- $I_G = 2 \times 3^3 / 12 = 4.5\ m^4$. $h^* = 1.5 + 4.5 / (6 \times 1.5) =$ 2.0 m below the surface (2/3 of depth).

➤ Buoyancy and stability

- **Archimedes' principle**: upward buoyant force = weight of fluid displaced. It acts at the **centre of buoyancy** B (centroid of displaced volume).
- **Metacentre** M: point about which a floating body starts to oscillate for small tilt.

FORMULA Metacentric height $GM = I / V - BG$ (I = MOI of water-line area about tilt axis, V = displaced volume).

Condition (floating body)	Equilibrium
M above G (GM > 0)	Stable
M at G (GM = 0)	Neutral
M below G (GM < 0)	Unstable

- For a fully submerged body, stable if B is above G.

3. Fluid Kinematics

- **Steady flow**: properties at a point do not change with time. **Uniform flow**: velocity does not change with position.
- **Laminar**: smooth layers. **Turbulent**: random mixing. Pipe flow: Re < 2000 laminar, Re > 4000 turbulent (approx).
- **Streamline**: tangent gives velocity direction. **Pathline**: path of one particle. **Streakline**: line of all particles that passed a point. In steady flow, all three are the same.

FORMULA Continuity (incompressible): A1 V1 = A2 V2 = Q. General 3D: $\partial u/\partial x + \partial v/\partial y + \partial w/\partial z = 0$.

➤ Stream function and velocity potential (common convention)

- Velocity potential ϕ: $u = \partial\phi/\partial x$, $v = \partial\phi/\partial y$. It exists only for irrotational flow.
- Stream function ψ: $u = \partial\psi/\partial y$, $v = -\partial\psi/\partial x$. It exists for any 2D incompressible flow.
- Lines of constant ϕ and constant ψ cut at right angles (flow net). Some books use the opposite sign; follow one convention.
- **Rotation** $\omega z = (1/2)(\partial v/\partial x - \partial u/\partial y)$. Vorticity = 2ω. Irrotational flow: $\omega z = 0$.

4. Fluid Dynamics

- **Euler's equation** (along a streamline): $dp/\rho + V\,dV + g\,dz = 0$.
- **Bernoulli's equation**: integrate Euler's for steady, incompressible, non-viscous flow along a streamline.

FORMULA $p/(\rho g) + V^2/(2g) + z$ = constant (pressure head + velocity head + datum head).

Flow measuring devices

Device	Principle	Formula
Venturimeter	Bernoulli + continuity	Q = Cd A1 A2 sqrt(2gh) / sqrt(A1^2 − A2^2)
Orifice meter	Same, cheaper, more loss	Same form, Cd ≈ 0.6–0.65
Pitot tube	Stagnation pressure	V = sqrt(2gh)
Notch / weir	Flow over an opening	Rectangular: Q = (2/3) Cd b sqrt(2g) H^(3/2)

- Differential manometer reading x: h = x (Sm/S − 1). For mercury–water, h = 12.6x.

EXAMPLE Venturimeter 200 mm x 100 mm, mercury manometer shows 200 mm, Cd = 0.98. Find Q.

- h = 0.2 x (13.6 − 1) = 2.52 m of water. sqrt(2gh) = sqrt(2 x 9.81 x 2.52) = 7.03 m/s.
- A2 = (π/4) x 0.1^2 = 0.007854 m^2, A2/A1 = 1/4.
- Q = 0.98 x 0.007854 x 7.03 / sqrt(1 − 1/16) = 0.98 x 0.0552 / 0.968 = 0.0559 m^3/s (55.9 L/s).

Momentum equation

- Net force = rate of change of momentum: ΣF = ρQ(V2 − V1). Used for forces on pipe bends, nozzles and jets.
- Jet on a fixed flat plate (normal): F = ρ a V^2. Moving plate: F = ρ a (V − u)^2.

5. Viscous (Laminar) Flow

FORMULA Hagen–Poiseuille (pipe): Δp = 32 μ V L / D^2. Max velocity = 2 x average velocity.

- Laminar flow between fixed parallel plates: Δp = 12 μ V L / t^2, Umax = 1.5 x average.
- Darcy friction factor for laminar pipe flow: f = 64 / Re.
- Shear stress varies linearly with radius: zero at centre, maximum at wall.

6. Flow Through Pipes

FORMULA Darcy-Weisbach: hf = f L V^2 / (2 g D) (f = Darcy factor; some books write 4f with Fanning f).

- **Moody chart**: f vs Re and relative roughness ε/D.

Minor loss	Head loss
Sudden expansion	(V1 − V2)^2 / (2g)
Sudden contraction	0.5 V2^2/(2g) approx.
Pipe entrance	0.5 V^2 / (2g)
Pipe exit	V^2 / (2g)

- **Pipes in series**: same Q, losses add. **Pipes in parallel**: same head loss, flows add.
- **Equivalent pipe** (Dupuit): L/D^5 = Σ Li / Di^5.
- Power through a pipe is maximum when hf = H/3; max efficiency = 66.7%.
- **Water hammer**: pressure rise when a valve is closed suddenly.
- **HGL** (hydraulic grade line) = p/ρg + z. **TEL** (total energy line) = HGL + V^2/2g.

EXAMPLE Water flows at 2 m/s in a 100 mm pipe, 500 m long, f = 0.02. hf = 0.02 x 500 x 2^2 / (2 x 9.81 x 0.1) = 40 / 1.962 = 20.4 m.

7. Boundary Layer and Dimensional Analysis

- **Boundary layer**: thin layer near a wall where velocity goes from 0 (no-slip) to free-stream U. Thickness δ is where u = 0.99U.
- **Displacement thickness** δ*: distance the wall would shift to give the same flow if there were no boundary layer. **Momentum thickness** θ: loss of momentum.

FORMULA Laminar flat plate (Blasius): δ / x = 5 / sqrt(Re_x). Shape factor H = δ*/θ (about 2.59 for Blasius).

- **Separation**: happens with adverse pressure gradient (dp/dx > 0). At separation (du/dy) at wall = 0.
- **Drag** = skin friction drag + pressure (form) drag. Lift acts normal to the flow.

➤ Dimensionless numbers

Number	Formula	Ratio
Reynolds Re	ρ V L / μ	Inertia / viscous
Froude Fr	V / sqrt(g L)	Inertia / gravity
Euler Eu	V / sqrt(p/ρ)	Inertia / pressure
Weber We	V / sqrt(σ/(ρL))	Inertia / surface tension
Mach M	V / c	Inertia / elastic

- **Buckingham π theorem**: with n variables and m fundamental dimensions, there are (n − m) dimensionless π terms.
- Model laws: pipes and submarines use Reynolds model law; ships, spillways and open channels use Froude model law.

8. Hydraulic Turbines

Feature	Pelton	Francis	Kaplan
Type	Impulse	Reaction	Reaction
Head	High	Medium	Low
Flow direction	Tangential	Radial inward / mixed	Axial
Discharge	Low	Medium	High
Specific speed (approx, metric)	10–35	60–300	300–1000

FORMULA Turbine specific speed Ns = N sqrt(P) / H^(5/4). Pump specific speed Ns = N sqrt(Q) / H^(3/4).

- **Pelton**: max hydraulic efficiency when bucket speed u = V/2 (half the jet speed). Jet ratio m = D/d (often about 12).
- **Kaplan**: adjustable runner blades, good efficiency at part load. **Propeller turbine** = fixed blades.
- **Draft tube** (reaction turbines only): lets the turbine sit above tail race and recovers exit KE.
- Efficiencies: hydraulic, mechanical, volumetric; overall = hydraulic x mechanical (x volumetric).

- **Governing**: Pelton uses a spear (needle) valve and deflector; reaction turbines use guide vane (wicket gate) adjustment.

9. Pumps

➤ Centrifugal pump

- Converts mechanical energy to pressure energy by a rotating impeller and casing (volute / diffuser).
- **Manometric head** Hm = head the pump actually gives. Manometric efficiency = g Hm / (Vw2 u2).
- **Priming**: fill the casing with liquid before starting, else the pump only churns air.
- Multistage: impellers in **series** for high head, in **parallel** for high discharge.
- **NPSH** (net positive suction head) available must be more than NPSH required to avoid cavitation.

➤ Reciprocating pump

- Positive displacement. Q_theory = A L N / 60 (single acting). **Slip** = (Qth − Qact)/Qth; negative slip can happen with long delivery pipe and high speed.
- **Air vessels** reduce acceleration head, give near-uniform flow and allow higher speed.

Point	Centrifugal	Reciprocating
Flow	Continuous, smooth	Pulsating
Best for	High Q, low–medium head	Low Q, high head
Priming	Needed	Not needed
Cost / maintenance	Low	Higher

10. Most Asked Questions (practice)

1. State Newton's law of viscosity. Explain Newtonian and non-Newtonian fluids.
2. Derive the expression for capillary rise.
3. Find total pressure and centre of pressure on an inclined plane surface.
4. Define metacentric height. Explain stability of floating bodies.
5. Explain stream function and velocity potential. Show that ϕ and ψ lines are orthogonal.
6. Derive Bernoulli's equation from Euler's equation. State its assumptions.

7. Derive discharge through a venturimeter.
8. Derive the Hagen–Poiseuille equation.
9. State Buckingham π theorem. Explain Reynolds and Froude model laws.
10. Compare Pelton, Francis and Kaplan turbines. Explain cavitation and NPSH.

TIP In GATE ME, focus on manometers, hydrostatic force, Bernoulli/venturi, laminar flow (Hagen–Poiseuille), Darcy losses, boundary layer, dimensional analysis and turbine velocity triangles.