Heat and Mass Transfer

Short notes for B.Tech Mechanical · GATE ME

Uncodemy Handwritten Notes · free for students

1. Modes of Heat Transfer

- Thermodynamics tells how much heat; heat transfer tells how fast (rate, in watts).
- Heat always flows from high temperature to low temperature.

Mode	How	Law
Conduction	Molecule to molecule, no bulk motion (solids)	Fourier: $q = -k\, dT/dx$
Convection	Fluid motion carries heat from a surface	Newton: $Q = h\, A\, (Ts - T\infty)$
Radiation	Electromagnetic waves, needs no medium	Stefan–Boltzmann: $Eb = \sigma T^4$

- **Thermal conductivity** k (W/m·K): metals high (copper about 385), insulators low (glass wool about 0.04).
- **Thermal diffusivity** $\alpha = k / (\rho\, c)$ (m^2/s): how fast temperature spreads in a body.

REMEMBER Silver > copper > aluminium > steel in conductivity. Pure metals conduct better than alloys.

2. Steady One-Dimensional Conduction

- **General conduction equation** (constant k): $\nabla^2 T + q_g/k = (1/\alpha)\, \partial T/\partial t$.
  - No generation, steady: Laplace equation $\nabla^2 T = 0$.
  - Steady with generation: Poisson equation. Unsteady, no generation: Fourier equation.

Thermal resistance (electrical analogy)

Geometry	Conduction resistance
Plane wall	$R = L / (kA)$
Hollow cylinder	$R = \ln(r2/r1) / (2\pi k L)$
Hollow sphere	$R = (r2 - r1) / (4\pi k\, r1\, r2)$
Convection film	$R = 1 / (hA)$

FORMULA $Q = \Delta T_{overall} / \Sigma R$. Layers in series: resistances add. Overall U: $1/(UA) = \Sigma R$.

EXAMPLE Furnace wall: brick 0.2 m ($k = 0.8$) + insulation 0.05 m ($k = 0.05$), area 1 m^2, inside 600°C, outside 50°C. Find Q and interface temperature.

- $R1 = 0.2/0.8 = 0.25$ K/W, $R2 = 0.05/0.05 = 1.0$ K/W, total = 1.25 K/W.
- $Q = (600 - 50) / 1.25 =$ 440 W.
- Interface $T = 600 - 440 \times 0.25 =$ 490°C.

Critical radius of insulation

- Adding insulation to a small pipe or wire can increase heat loss first, because outer surface area grows.

FORMULA Critical radius: cylinder $r_c = k / h$, sphere $r_c = 2k / h$. Heat loss is maximum at $r = r_c$.

- Electric cables: keep insulation radius near r_c to cool better. Steam pipes: keep r well above r_c to reduce loss.

Heat generation

- Plane wall (thickness 2L, both faces at Ts): $T_{max} = T_s + q_g L^2 / (2k)$, at the centre.
- Solid cylinder radius R: $T_{max} = T_s + q_g R^2 / (4k)$.

3. Extended Surfaces (Fins)

- Fins increase surface area to raise heat transfer by convection (bike engines, radiators, transformers).

FORMULA $m = \sqrt{h P / (k A)}$. Long fin: $Q = \sqrt{h P k A}\ \theta 0$. Insulated tip: $Q = \sqrt{h P k A}\ \theta 0 \tanh(mL)$. ($\theta 0 = T_{base} - T_\infty$)

- **Fin efficiency** η = actual heat / heat if the whole fin were at base temperature. Insulated tip: $\eta = \tanh(mL) / (mL)$.
- **Fin effectiveness** ε = heat with fin / heat without fin. A fin is worth using only if $\varepsilon > 2$ (roughly).
- Fins work best with low h (gas side), high k material, thin and closely spaced.
- Corrected length for a convecting tip: $L_c = L + t/2$ (rectangular fin).

4. Transient (Unsteady) Conduction

- **Biot number** $Bi = h L_c / k$ = internal (conduction) resistance / external (convection) resistance.
- **Lumped system analysis** is valid when $Bi < 0.1$: temperature is uniform inside the body at any time.

FORMULA $(T - T_\infty)/(T_i - T_\infty) = \exp(-h A_s t / (\rho V c))$. Time constant $\tau = \rho V c / (h A_s)$. $L_c = V / A_s$.

- L_c: plate (thickness 2L) = L, long cylinder = R/2, sphere = R/3.
- **Fourier number** $Fo = \alpha t / L_c^2$ (dimensionless time). Exponent = $-Bi \times Fo$.
- For $Bi > 0.1$, use **Heisler charts** or series solutions.
- Thermocouple response: small bead, high h gives a short time constant (fast response).

5. Convection

- **Forced convection**: flow by a fan or pump. **Free (natural) convection**: flow due to buoyancy (density difference).

Number	Formula	Meaning
Nusselt Nu	h L / k	Convection / conduction in the fluid
Prandtl Pr	μ cp / k = ν / α	Momentum diffusivity / heat diffusivity
Reynolds Re	ρ V L / μ	Inertia / viscous (forced)
Grashof Gr	g β ΔT L^3 / ν^2	Buoyancy / viscous (free)
Rayleigh Ra	Gr x Pr	Decides laminar or turbulent in free convection
Stanton St	Nu / (Re Pr)	Heat transferred / heat capacity of flow

- Forced: Nu = f(Re, Pr). Free: Nu = f(Gr, Pr).
- **Thermal boundary layer** δt vs velocity layer δ: δ/δt ≈ Pr^(1/3). For Pr = 1 they are equal; liquid metals Pr << 1 (δt much thicker); oils Pr >> 1.

➤ Useful correlations

- Flat plate, laminar: local Nu_x = 0.332 Re_x^0.5 Pr^(1/3); average Nu = 0.664 Re_L^0.5 Pr^(1/3).
- Laminar fully developed tube flow: Nu = 3.66 (constant wall temperature), Nu = 4.36 (constant heat flux).
- Turbulent tube flow (Dittus–Boelter): Nu = 0.023 Re^0.8 Pr^n, n = 0.4 for heating the fluid, 0.3 for cooling.
- **Reynolds analogy**: St = f/8 (Darcy f) for Pr = 1. Colburn analogy extends it: St Pr^(2/3) = f/8.

> **TIP** In convection numericals: find fluid properties at film temperature (Ts + T∞)/2, find Re, choose the right correlation, find Nu, then h = Nu k / L.

6. Boiling and Condensation

- **Pool boiling curve** (heat flux vs excess temperature ΔTe = Ts − Tsat):
  1. Free convection boiling
  2. **Nucleate boiling** (bubbles, very high h, ends at the critical heat flux)
  3. Transition boiling (unstable)
  4. **Film boiling** (vapour blanket, beyond the Leidenfrost point)
- Equipment is designed to work in nucleate boiling, below critical heat flux, to avoid burnout.

- **Condensation**: filmwise (liquid film covers surface) and dropwise (drops roll off). Dropwise gives a much higher h but is hard to keep going.
- Nusselt's theory gives h for laminar film condensation on vertical plates and tubes.

7. Radiation

- All bodies above 0 K emit radiation. Thermal radiation wavelength range ≈ 0.1 to 100 μm.
- For incident radiation: α + ρ + τ = 1 (absorptivity, reflectivity, transmissivity). Opaque body: τ = 0.
- **Black body**: absorbs all radiation (α = 1), best emitter. **Grey body**: ε is the same at all wavelengths (ε < 1).

FORMULA Stefan-Boltzmann: Eb = σT^4, σ = 5.67 x 10^-8 W/m^2·K^4. Real surface: E = ε σ T^4.

FORMULA Wien's displacement law: λmax x T = 2898 μm·K.

- **Kirchhoff's law**: at thermal equilibrium, emissivity = absorptivity (ε = α).
- **Planck's law** gives the spectral distribution of black body emission.

EXAMPLE Black body at 1000 K: Eb = 5.67 x 10^-8 x 1000^4 = 56.7 kW/m^2; peak wavelength = 2898/1000 = 2.9 μm.

View factor (shape factor)

- F12 = fraction of radiation leaving surface 1 that reaches surface 2.
- **Reciprocity**: A1 F12 = A2 F21. **Summation**: ΣFij = 1. Flat or convex surface: F11 = 0.

FORMULA Two large parallel grey plates: q = σ(T1^4 − T2^4) / (1/ε1 + 1/ε2 − 1).

- **Radiation shields**: N shields of the same emissivity as the plates cut heat transfer to 1/(N + 1).
- Small body in a large enclosure: Q = ε1 σ A1 (T1^4 − T2^4).

8. Heat Exchangers

Type	Flow arrangement	Note
Parallel flow	Both fluids same direction	Cold exit always below hot exit
Counter flow	Opposite directions	Most effective; cold exit can exceed hot exit
Cross flow	At right angles	Car radiator, air coolers
Shell and tube	One fluid in tubes, other in shell	Power plants, refineries

FORMULA LMTD = (ΔT1 − ΔT2) / ln(ΔT1/ΔT2). Q = U A LMTD (x correction factor F for multipass / cross flow).

EXAMPLE Counter flow: hot oil 150 → 90°C, water 30 → 70°C, U = 500 W/m^2·K, Q = 50 kW. Find area.

- ΔT1 = 150 − 70 = 80, ΔT2 = 90 − 30 = 60.
- LMTD = (80 − 60) / ln(80/60) = 20 / 0.2877 = 69.5°C.
- A = 50000 / (500 x 69.5) = 1.44 m^2.

➤ NTU–effectiveness method

- Used when exit temperatures are not known.
- C = m cp (heat capacity rate). Cmin and Cmax = smaller and larger of the two. NTU = U A / Cmin.
- Effectiveness ε = Q / Qmax, with Qmax = Cmin (Th,in − Tc,in).
- For condensers and evaporators (Cr = Cmin/Cmax = 0): ε = 1 − e^(−NTU), same for all flow arrangements.
- **Fouling factor**: extra resistance due to scale and deposits; lowers U over time.

9. Mass Transfer

- Mass moves from high concentration to low concentration (like heat from hot to cold).

FORMULA Fick's law: J_A = −D_AB dC_A/dx (D_AB = mass diffusivity, m^2/s).

Number	Formula	Heat transfer analogue
Schmidt Sc	ν / D	Prandtl Pr
Sherwood Sh	hm L / D	Nusselt Nu
Lewis Le	α / D = Sc / Pr	Links heat and mass transfer

- Examples: evaporation of water into air, drying, absorption of gases, wet-bulb thermometer.

10. Most Asked Questions (practice)

1. Derive the general 3D heat conduction equation in Cartesian coordinates.
2. Find heat flow through a composite wall or composite cylinder.
3. Derive the critical radius of insulation for a cylinder.
4. Derive the temperature distribution and heat flow for a fin with insulated tip.
5. Explain lumped system analysis. When is it valid?
6. Explain the physical meaning of Nu, Pr, Re and Gr.
7. Draw and explain the pool boiling curve.
8. State Stefan–Boltzmann, Kirchhoff and Wien's laws. Explain view factor rules.
9. Derive LMTD for a parallel flow heat exchanger.
10. Explain the NTU–effectiveness method with a solved example.

TIP In GATE ME, the most common numericals are composite walls, critical radius, fins, lumped capacity, radiation between surfaces and LMTD / NTU heat exchangers. Learn the resistance table by heart.