Industrial Engineering and Operations Research

Short notes for B.Tech Mechanical · GATE ME

Uncodemy Handwritten Notes · free for students

I. Introduction, Productivity and Plant Layout

- **Industrial engineering** = designing and improving systems of people, machines, materials and money to work efficiently.
- **Productivity** = output / input. Raise it by better methods, less waste, better planning.

➤ Plant layout types

Layout	Arrangement	Best for
Product (line)	Machines in order of operations	Mass production, few products (cars, bottling)
Process (functional)	Similar machines grouped	Job shops, many varied products
Fixed position	Product stays, men and machines come to it	Ships, aircraft, buildings
Cellular (group technology)	Cells for part families	Medium variety, medium volume

- Plant location factors: raw material, market, labour, power, water, transport, government policy.
- **Break-even analysis**: total cost = fixed cost + variable cost x quantity.

FORMULA Break-even quantity $Q = F / (S - V)$ (F = fixed cost, S = selling price per unit, V = variable cost per unit).

EXAMPLE F = Rs 1,00,000, S = Rs 50, V = Rs 30. Q = 1,00,000 / 20 = 5000 units.

2. Work Study

- Work study = **method study** (find the best way to do a job) + **work measurement** (find the time it should take).

➤ Method study

- Steps (SREDIM): Select, Record, Examine, Develop, Install, Maintain.
- Process chart symbols (ASME):

Symbol	Activity
Circle	Operation
Arrow	Transport
Square	Inspection
D shape	Delay (temporary storage)
Triangle	Storage

- Charts: outline process chart, flow process chart, two-handed chart, multiple activity chart, flow diagram, string diagram, SIMO chart (with therbligs, used in micro-motion study).
- **Principles of motion economy**: both hands should start and end together, use smooth curved motions, keep tools in fixed nearby places.

➤ Work measurement

- Methods: time study (stopwatch), work sampling, PMTS (MTM, work factor), standard data.

FORMULA Normal time = observed time x rating / 100. Standard time = normal time x (1 + allowance fraction).

EXAMPLE Observed time 2.0 min, rating 110%, allowance 15%. Normal time = 2.2 min. Standard time = 2.2 x 1.15 = 2.53 min.

- **Allowances**: personal needs, fatigue, delays (contingency).

3. Forecasting

- Qualitative: Delphi method, market survey, expert opinion. Quantitative: time series, causal (regression).

Method	Formula / idea
Simple moving average	Average of the last n periods
Weighted moving average	Recent periods get more weight
Exponential smoothing	$F(t+1) = F(t) + \alpha (D(t) - F(t))$
Linear regression	$y = a + b x$ (least squares)

- α close to 1: reacts quickly to change. α close to 0: smooth, slow reaction.
- Errors: MAD = $\Sigma|\text{error}| / n$, MSE = Σ error^2 / n, bias = Σ error / n. **Tracking signal** = running sum of errors / MAD.

EXAMPLE Forecast $F(t) = 100$, actual $D(t) = 120$, $\alpha = 0.2$. $F(t+1) = 100 + 0.2 \times 20 =$ 104.

4. Inventory Control

- Inventory costs: ordering (setup) cost S per order, holding (carrying) cost H per unit per year, shortage cost, purchase cost.

FORMULA EOQ (Wilson) $Q^* = \text{sqrt}(2\ D\ S / H)$. At EOQ, annual ordering cost = annual holding cost.

FORMULA Total cost $TC = (D/Q)\ S + (Q/2)\ H$. Production (EPQ) model: $Q^* = \text{sqrt}(2\ D\ S / (H\ (1 - d/p)))$.

EXAMPLE D = 10000 units/year, S = Rs 200 per order, H = Rs 4 per unit per year.

- $Q^* = \text{sqrt}(2 \times 10000 \times 200 / 4) = \text{sqrt}(10{,}00{,}000) =$ 1000 units.
- Orders per year = 10. Ordering cost = 10 x 200 = Rs 2000. Holding cost = 500 x 4 = Rs 2000.
- Total (ordering + holding) = Rs 4000.
- **Reorder point** ROP = demand during lead time + safety stock = d x L + SS.
- With shortages allowed (backorders), EOQ becomes larger: $Q^* = \text{sqrt}(2DS/H) \times \text{sqrt}((H + B)/B)$, B = backorder cost.

➤ Selective inventory control

Method	Basis	Groups
ABC	Annual consumption value	A: ~10% items, ~70% value; C: many items, little value
VED	Criticality	Vital, Essential, Desirable
FSN	Movement	Fast, Slow, Non-moving
HML	Unit price	High, Medium, Low

- **MRP** (material requirements planning): uses master production schedule + bill of materials + inventory records to plan orders for dependent-demand items.
- **JIT** (just in time): produce only what is needed, when needed; Kanban cards; aims at zero inventory.

5. Production Planning, Scheduling and Line Balancing

- Aggregate planning strategies: chase demand (vary output), level production (use inventory), vary workforce, overtime, subcontracting.

➤ Sequencing rules (single machine)

Rule	Gives
SPT (shortest processing time)	Minimum mean flow time and mean waiting
EDD (earliest due date)	Minimum maximum lateness
FCFS	Fair, simple; not optimal

➤ Johnson's rule (n jobs, 2 machines, same order)

1. Pick the smallest time in the list.
2. If it is on machine 1, put that job as early as possible; if on machine 2, as late as possible.
3. Remove the job and repeat.

EXAMPLE Jobs A–E, times (M1, M2): A (5, 2), B (1, 6), C (9, 7), D (3, 8), E (10, 4).

- Smallest = 1 (B, M1) → first. Next 2 (A, M2) → last. Next 3 (D, M1) → second. Next 4 (E, M2) → fourth. C in the middle.
- Sequence: B – D – C – E – A.
- M1 ends: B 1, D 4, C 13, E 23, A 28. M2 ends: B 7, D 15, C 22, E 27, A 30.
- Makespan = 30 hours.

➤ Line balancing

FORMULA Cycle time C = available time / required output. Min stations = Σ task times / C. Line efficiency = Σ t / (n x C).

- **Balance delay** = 1 – line efficiency. Heuristics: largest candidate rule, ranked positional weight method.

6. Quality Control

- **SQC**: control charts (process control) + acceptance sampling (product control).

Chart	For	Limits (centre ± 3σ)
X-bar	Mean of variables	$\bar{\bar{X}} \pm A2\ \bar{R}$
R	Range of variables	$LCL = D3\ \bar{R}$, $UCL = D4\ \bar{R}$
p	Fraction defective	$\bar{p} \pm 3\ sqrt(\bar{p}(1-\bar{p})/n)$
c	Defects per unit	$\bar{c} \pm 3\ sqrt(\bar{c})$

- Chance (common) causes vs assignable (special) causes. Points outside limits mean assignable causes.
- **Process capability** Cp = (USL – LSL) / (6σ). Cp >= 1.33 is usually considered capable.
- **Acceptance sampling**: OC curve, AQL, LTPD; producer's risk α (good lot rejected), consumer's risk β (bad lot accepted).
- **Six Sigma**: DMAIC (Define, Measure, Analyse, Improve, Control); target 3.4 defects per million opportunities. **TQM**, Kaizen, 5S, Poka-yoke.

7. Linear Programming

- LP = maximise or minimise a linear objective subject to linear constraints and x >= 0.

- **Graphical method** (2 variables): draw constraints, find the feasible region; the optimum lies at a corner point.
- **Simplex method**: add slack (for <=), subtract surplus and add artificial (for >=); use Big-M or two-phase method for artificials.
- Special cases: unbounded solution, infeasible, multiple optima (non-basic variable has zero in the objective row at optimum), degeneracy (a basic variable = 0).
- **Duality**: every LP (primal) has a dual. Max primal <→ min dual; primal constraints become dual variables. At optimum, both objective values are equal. **Shadow price** = value of one more unit of a resource.

8. Transportation and Assignment

➤ Transportation problem

- m sources, n destinations. Balanced if total supply = total demand; else add a dummy row or column with zero cost.
- Initial solution: North-West Corner, Least Cost, Vogel's Approximation Method (VAM) (usually closest to optimum).
- Basic feasible solution has $m + n - 1$ allocations; fewer = degenerate (add a small ε).
- Optimality test: **MODI (u–v) method** or stepping stone. $u_i + v_j = c_{ij}$ for allocated cells; all $\Delta_{ij} = c_{ij} - u_i - v_j >= 0$ for a minimum.

➤ Assignment problem

- n jobs to n workers, one each, min total cost. Solved by the **Hungarian method**:
  1. Subtract row minimum from each row, then column minimum from each column.
  2. Cover all zeros with the fewest lines. If lines = n, make the assignment.
  3. Else subtract the smallest uncovered value from uncovered cells, add it at intersections, repeat.
- Maximisation: subtract all values from the largest first. Unbalanced: add a dummy row or column.

9. Queuing Theory and PERT/CPM

➤ M/M/1 queue (Poisson arrivals λ, exponential service μ, one server)

FORMULA $\rho = \lambda/\mu$, $P0 = 1 - \rho$, $Ls = \lambda/(\mu - \lambda)$, $Lq = \lambda^2/(\mu(\mu - \lambda))$, $Ws = 1/(\mu - \lambda)$, $Wq = \lambda/(\mu(\mu - \lambda))$.

- **Little's law**: $L = \lambda W$ (in system and in queue).

EXAMPLE $\lambda = 8$ per hour, $\mu = 10$ per hour. $\rho = 0.8$, $P0 = 0.2$. $Ls = 8/2 =$ 4 customers. $Lq = 64/20 = 3.2$. $Ws = 1/2$ h = 30 min. $Wq = 8/20$ h = 24 min.

➤ PERT and CPM

Point	CPM	PERT
Time estimate	One (deterministic)	Three (probabilistic)
Focus	Time–cost trade-off (crashing)	Time uncertainty
Used for	Repetitive projects (construction)	R&D, new projects

FORMULA PERT expected time $te = (a + 4m + b)/6$, variance $\sigma^2 = ((b - a)/6)^2$. $Z = (T - Te)/\sigma$ (σ of the critical path).

EXAMPLE $a = 2$, $m = 5$, $b = 14$ days. $te = (2 + 20 + 14)/6 =$ 6 days; $\sigma = 12/6 = 2$ days, variance = 4.

- **Critical path**: longest path through the network; its activities have zero total float.
- Total float = LS − ES = LF − EF. Free float = ES of next activity − EF of this activity.
- Project variance = sum of variances of critical activities.
- **Crashing**: shorten critical activities with the lowest cost slope first; cost slope = (crash cost − normal cost)/(normal time − crash time).

10. Most Asked Questions (practice)

1. Compare product, process and fixed-position layouts.
2. Explain method study steps and process chart symbols.

3. Find standard time from given observed time, rating and allowances.
4. Forecast using moving average and exponential smoothing; find MAD.
5. Derive the EOQ formula and solve a numerical.
6. Explain ABC and VED analysis.
7. Sequence jobs on two machines by Johnson's rule; find makespan and idle time.
8. Construct X-bar, R and p charts from given data.
9. Solve an LP graphically and by simplex. Write its dual.
10. Find critical path, floats and the probability of completing a project by a given time.

TIP In GATE ME, IE/OR is very scoring: EOQ, forecasting (exponential smoothing), M/M/1 queue, PERT/CPM, Johnson's rule, LP graphical and break-even come almost every year. Practise them for speed.