Machine Design

Short notes for B.Tech Mechanical · GATE ME

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1. Design Basics

- **Machine design** = choosing materials, shapes and sizes of parts so they work safely, last long and cost little.
- Steps: need → synthesis → analysis of forces → material selection → sizing → modification → drawing → production.
- **Factor of safety (FOS)** = failure stress / working (allowable) stress.
  - Ductile material: failure stress = yield strength Syt. Brittle material: failure stress = ultimate strength Sut.
- **Preferred numbers** (Renard series R5, R10, R20, R40) are used for standard sizes, e.g. R10 ratio = 10^(1/10) ≈ 1.26.

➤ Material properties

Property	Meaning
Strength	Ability to resist load without failure
Stiffness	Ability to resist deformation (modulus E)
Ductility	Large plastic strain before fracture (elongation > 5%)
Toughness	Energy absorbed before fracture (area under σ–ε curve)
Resilience	Energy stored in the elastic range
Hardness	Resistance to indentation / scratching

- Designations (Indian, IS): plain carbon steel 40C8 = 0.40% carbon (0.35–0.45), 0.8% manganese (approx.). Grey cast iron FG 200 = min tensile strength 200 MPa.

2. Theories of Failure (static loading)

Theory	Failure when	Suitable for
Max principal stress (Rankine)	σ1 = Sut / N	Brittle materials
Max shear stress (Tresca / Guest)	τmax = Syt / (2N)	Ductile, conservative
Max distortion energy (von Mises)	σ1^2 + σ2^2 − σ1σ2 = (Syt/N)^2	Ductile, most accurate
Max principal strain (St. Venant)	Max strain = yield strain	Rarely used
Max strain energy (Haigh)	Total strain energy = value at yield	Rarely used

- In pure shear: Tresca gives Ssy = 0.5 Syt; von Mises gives Ssy = 0.577 Syt.
- Tresca hexagon lies inside the von Mises ellipse, so Tresca is safer (more conservative).

FORMULA Principal stresses (2D): σ1,2 = (σx + σy)/2 ± sqrt(((σx − σy)/2)^2 + τxy^2). τmax = (σ1 − σ2)/2.

3. Fatigue and Stress Concentration

- **Fatigue**: failure under repeated (cyclic) stress, at stress well below the static strength. It starts as a tiny crack and grows.
- **Endurance limit** Se: stress below which the part lasts an infinite number of cycles (steel has one, aluminium does not).
- For steel specimens: Se' ≈ 0.5 Sut. Actual part: Se = Ka Kb Kc Kd x Se' (surface, size, reliability, other factors).
- Mean stress σm = (σmax + σmin)/2. Amplitude σa = (σmax − σmin)/2.

FORMULA Soderberg: σa/Se + σm/Syt = 1/N. Goodman: σa/Se + σm/Sut = 1/N. Gerber: N σa/Se + (N σm/Sut)^2 = 1.

- Soderberg is the most conservative (safest), Gerber the least. Goodman is commonly used.

Stress concentration

- Sudden change in section (holes, grooves, fillets, keyways) raises local stress.
- **Theoretical factor** Kt = max local stress / nominal stress (depends only on geometry).
- **Fatigue factor** Kf; **notch sensitivity** q.

FORMULA Kf = 1 + q (Kt − 1). q = 0: no notch effect; q = 1: full effect (Kf = Kt).

- Reduce it: generous fillet radius, relief grooves, avoid sharp corners, shot peening, polishing.

4. Shafts, Keys and Couplings

Shaft design (ASME / combined loading)

FORMULA Equivalent torque Te = sqrt(M^2 + T^2) (use with τ). Equivalent moment Me = (M + sqrt(M^2 + T^2))/2 (use with σ).

FORMULA Solid shaft: τ = 16 Te / (π d^3), σb = 32 Me / (π d^3). Power P = 2π N T / 60.

EXAMPLE A shaft carries M = 1 kN·m and T = 1.5 kN·m; allowable τ = 50 MPa. Find diameter.

- Te = sqrt(1^2 + 1.5^2) = 1.803 kN·m = 1.803 x 10^6 N·mm.
- d^3 = 16 x 1.803 x 10^6 / (π x 50) = 1.836 x 10^5 mm^3.
- d = 56.8 mm; choose a standard size of 60 mm.
- Design for rigidity: angle of twist θ = T L / (G J), J = π d^4 / 32. Limit is often 0.25° per metre length.
- Hollow shafts are stronger and stiffer per kg than solid shafts.

Keys

- Types: sunk (rectangular, square), saddle, feather, Woodruff, splines. Key is weaker than the shaft so it fails first (and is cheap to replace).
- Rectangular sunk key (width w, height h, length l) under torque T:

- Shear: $\tau = 2T / (w \times d)$. Crushing: $\sigma_c = 4T / (h \times d)$.
- For a square key ($w = h$), the key is equally strong in shear and crushing when $\sigma_c = 2\tau$.

➤ Couplings

Coupling	Allows misalignment?	Use
Muff / sleeve	No	Light duty, aligned shafts
Rigid flange	No	Aligned shafts, medium power
Bushed pin flexible	Small	Motor to pump, absorbs shock
Oldham	Parallel offset	Shafts with lateral offset
Universal (Hooke's)	Angular	Vehicle propeller shafts

5. Joints: Riveted, Welded, Bolted

➤ Riveted joints

- Types: lap and butt (single / double strap). Failures: tearing of plate, shearing of rivet, crushing of plate or rivet.
  - Tearing: $P_t = (p - d)\, t\, \sigma_t$. Shearing: $P_s = n\, (\pi/4)\, d^2\, \tau$ (× 2 for double shear; Indian Boiler Regulations use 1.875). Crushing: $P_c = n\, d\, t\, \sigma_c$.

FORMULA Joint efficiency η = least of $(P_t, P_s, P_c) / (p\, t\, \sigma_t)$, where p = pitch, d = rivet hole diameter.

➤ Welded joints

- **Fillet weld**: throat $t = 0.707 \times$ leg size s.
  - Transverse fillet: $P = 0.707\, s \times \sigma_t$ (per weld). Parallel fillet: $P = 0.707\, s \times \tau$ (per weld).
- **Butt weld** strength: $P = t \times \sigma_t$ (t = plate thickness).
- Eccentric loading: add direct shear and torsional (or bending) shear as vectors.

➤ Bolted joints

- Bolt is designed on its **core (stress) area**, not on the nominal diameter.
- Initial tightening load (Indian practice, empirical): $P_i = 2840\, d$ N (d in mm) for fluid-tight joints.

- Bolts of uniform strength: reduce shank to core diameter, or drill an axial hole, to absorb shock better.

6. Springs

- Helical compression springs: spring index C = D/d (D = mean coil diameter, d = wire diameter). Usual C = 4 to 12.

FORMULA Shear stress $\tau = K \times 8 P D / (\pi d^3)$, Wahl factor $K = (4C - 1)/(4C - 4) + 0.615/C$.

FORMULA Deflection $\delta = 8 P D^3 n / (G d^4)$. Stiffness $k = G d^4 / (8 D^3 n)$.

- Springs in series: $1/k = 1/k1 + 1/k2$. In parallel: $k = k1 + k2$.
- Wahl factor includes direct shear and curvature effect.
- **Leaf springs** (vehicles): stress $\sigma = 6 P L / (n b t^2)$ (for cantilever-type leaf), nipping gives equal stress in full and graduated leaves.
- **Surge**: resonance of the spring coils; keep natural frequency at least 15–20 times the operating frequency.

EXAMPLE Spring: P = 300 N, D = 40 mm, d = 5 mm, n = 10 active coils, G = 80 GPa. Find τ and δ.

- $C = 40/5 = 8$. $K = 31/28 + 0.615/8 = 1.107 + 0.077 = 1.184$.
- $\tau = 1.184 \times 8 \times 300 \times 40 / (\pi \times 125) = 1.184 \times 244.5 =$ 289 MPa.
- $\delta = 8 \times 300 \times 40^3 \times 10 / (80000 \times 5^4) = 1.536 \times 10^9 / (5 \times 10^7) =$ 30.7 mm. Stiffness = 9.77 N/mm.

7. Bearings

➤ Rolling contact bearings

FORMULA Rated life $L10 = (C/P)^p$ million revolutions; p = 3 for ball, 10/3 for roller bearings. Equivalent load $P = X Fr + Y Fa$.

- C = dynamic load rating (load for 1 million revolutions life with 90% reliability). C0 = static rating.

- L10 in hours = L10 x 10^6 / (60 N).

EXAMPLE Ball bearing, C = 30 kN, P = 5 kN, N = 1000 rpm. L10 = (30/5)^3 = 216 million rev = 216 x 10^6 / (60 x 1000) = 3600 hours.

- If load is doubled, ball bearing life falls to 1/8.

➤ Sliding contact (journal) bearings

- **Hydrodynamic lubrication**: shaft rotation drags oil into a wedge and builds pressure (thick film).
- **Sommerfeld number** $S = (r/c)^2 \times \mu N / p$ (N in rev/s, p = load per projected area). Used in Raimondi–Boyd charts.
- **Petroff's equation**: friction in a lightly loaded bearing, coefficient of friction $f = 2\pi^2 (\mu N / p)(r / c)$.
- **Stribeck curve** (f vs μN/p): boundary → mixed → hydrodynamic. Minimum friction in the mixed zone; design is kept in the hydrodynamic zone.

8. Gears, Clutches, Brakes and Power Screws

➤ Gear design

FORMULA Lewis equation: beam strength $S_b = \sigma_b\, b\, m\, Y$ (Y = Lewis form factor). Dynamic effect: Barth velocity factor $C_v = 3/(3 + v)$ for ordinary cut gears.

- Face width b usually 10 m. Pinion is weaker (smaller Y), so design the pinion unless the gear material is weaker.
- **Buckingham** equation gives dynamic load; **wear strength** (Buckingham) checks surface pitting.

Clutches

Assumption	Torque per friction surface	Use
Uniform pressure	$T = (2/3)\ \mu\ W\ (R1^3 - R2^3)/(R1^2 - R2^2)$	New clutch
Uniform wear	$T = \mu\ W\ (R1 + R2)/2$	Worn clutch, safer design

- Uniform wear gives a lower torque, so it is used for design. Multiply by number of friction surfaces.

Brakes

- Block (shoe) brake, band brake ($T1/T2 = e^{(\mu\theta)}$), band and block, internal expanding shoe (vehicles), disc brake.
- Brake heat = kinetic + potential energy absorbed; check temperature rise.
- **Self-energising**: friction moment helps the applied force. **Self-locking**: no force needed (avoid in brakes).

Power screws

- Thread forms: square (most efficient), Acme / trapezoidal (easy to make, used in lead screws), buttress (load in one direction).

FORMULA Efficiency of square thread $\eta = \tan\alpha / \tan(\alpha + \phi)$ (α = helix angle, ϕ = friction angle). Max $\eta = (1 - \sin\phi)/(1 + \sin\phi)$.

- **Self-locking** when $\phi >= \alpha$; then efficiency is less than 50%. Screw jacks are made self-locking.

9. Most Asked Questions (practice)

1. Explain factor of safety. How is it chosen for ductile and brittle materials?
2. Compare theories of failure. Which is used for ductile parts and why?
3. Explain S–N curve, endurance limit and Soderberg / Goodman lines.
4. Define stress concentration and notch sensitivity. How can they be reduced?
5. Design a shaft under combined bending and torsion.
6. Find the efficiency of a riveted lap / butt joint.
7. Find the strength of an eccentrically loaded fillet weld.

8. Derive the Wahl factor based stress and deflection of a helical spring.
9. Select a ball bearing for given radial and axial loads and life.
10. Derive torque for a plate clutch under uniform pressure and uniform wear.

TIP In GATE ME, the scoring topics are failure theories, fatigue lines, bolt/weld/rivet strength, bearing life, clutches and brakes, and spring formulas. Keep standard formulas and units (N, mm, MPa) consistent.