Network Analysis

Short notes for B.Tech ECE · EEE · GATE EC

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1. Basic Concepts

- **Network** = interconnection of elements (R, L, C, sources). **Circuit** = a network with at least one closed path.
- **Active elements** supply energy (voltage source, current source). **Passive elements** use or store it (R, L, C).
- **Linear** element: V–I relation is a straight line (R, L, C). **Bilateral**: same behaviour in both directions (resistor). A diode is non-linear and unilateral.

➤ Element relations

Element	V–I relation	Energy	Behaviour at DC (steady state)
Resistor R	v = R i	Dissipated, P = i^2 R	Same as R
Inductor L	v = L di/dt	Stored, W = (1/2) L i^2	Short circuit
Capacitor C	i = C dv/dt	Stored, W = (1/2) C v^2	Open circuit

REMEMBER Inductor current and capacitor voltage cannot change suddenly (they are continuous at t = 0).

➤ Source types

- **Ideal voltage source**: zero internal resistance. **Ideal current source**: infinite internal resistance.
- **Dependent sources**: VCVS, VCCS, CCVS, CCCS (value controlled by another voltage or current in the circuit).
- **Source transformation**: voltage source V with series R = current source I = V/R with parallel R.

2. Network Laws and Analysis Methods

- **KCL**: sum of currents entering a node = sum leaving (charge conservation).

- **KVL**: sum of voltages around a closed loop = 0 (energy conservation).
- **Voltage divider**: V1 = V x R1 / (R1 + R2). **Current divider**: I1 = I x R2 / (R1 + R2).

Nodal vs mesh analysis

Point	Nodal analysis	Mesh analysis
Based on	KCL	KVL
Unknowns	Node voltages	Mesh currents
Number of equations	n − 1 (n = nodes)	b − n + 1 (b = branches)
Best when	Many current sources, few nodes	Many voltage sources, planar circuit
Special case	**Supernode** (voltage source between 2 nodes)	**Supermesh** (current source shared by 2 meshes)

Star–delta conversion

FORMULA Delta to star: Ra = Rab x Rca / (Rab + Rbc + Rca) (product of the two adjacent sides / sum).

FORMULA Star to delta: Rab = (Ra Rb + Rb Rc + Rc Ra) / Rc (sum of pair products / opposite resistor).

- For equal resistors: R(star) = R(delta) / 3.

3. Network Theorems

Theorem	Statement in short	Applies to
Superposition	Response = sum of responses to each independent source acting alone	Linear circuits only
Thevenin	Any linear 2-terminal network = Vth in series with Rth	Linear
Norton	Same network = IN in parallel with RN (RN = Rth, IN = Vth/Rth)	Linear
Maximum power transfer	Load gets max power when RL = Rth (AC: ZL = Zth*)	Linear
Reciprocity	Source and ammeter can swap places, reading stays same	Linear, bilateral, single source
Tellegen	Sum of v x i over all branches = 0	Any lumped network (KCL + KVL)
Millman	Parallel voltage sources with series R → one equivalent source	Linear
Substitution	A branch can be replaced by a source of the same V or I	Any

- In superposition: kill a voltage source = short it, kill a current source = open it. Dependent sources are never killed.
- Superposition does **not** apply to power (power is not linear in current).
- **Finding Rth with dependent sources**: Rth = Voc / Isc, or apply a test source and find Vtest / Itest.

FORMULA Pmax = Vth^2 / (4 Rth). Efficiency at maximum power transfer = 50%.

➤ Solved example (Thevenin + max power)

- A 12 V source in series with 4 Ω; a 6 Ω resistor is across terminals a–b. Find the Thevenin equivalent and max power.
  - Vth = open-circuit voltage = 12 x 6 / (4 + 6) = 7.2 V
  - Rth = 4 || 6 = (4 x 6) / 10 = 2.4 Ω (source shorted)
  - For max power, RL = 2.4 Ω. Pmax = 7.2^2 / (4 x 2.4) = 51.84 / 9.6 = 5.4 W

EXAM POINT For an AC circuit, max power needs ZL = complex conjugate of Zth. If only RL can vary, take RL = |Zth|.

4. Transient Analysis

- **Transient** = the response while the circuit moves from one steady state to another (after a switch operates).
- Total response = natural (source-free) response + forced (steady-state) response.

➤ First-order circuits

Circuit	Time constant τ	Response
RC	$\tau = R\,C$	$v(t) = v(\infty) + [v(0) - v(\infty)]\,e^{(-t/\tau)}$
RL	$\tau = L / R$	$i(t) = i(\infty) + [i(0) - i(\infty)]\,e^{(-t/\tau)}$

- At $t = \tau$ the charging value reaches **63.2%** of the final value. After **5τ** it is about 99.3% (taken as steady state).

FORMULA Any first-order quantity: $x(t) = x(\infty) + [x(0+) - x(\infty)]\,e^{(-t/\tau)}$, with $\tau = R_{eq}\,C$ or L / R_{eq}.

➤ Solved example (RC charging)

- R = 10 kΩ, C = 100 μF, a 10 V DC source is switched on at t = 0, capacitor uncharged.
  - $\tau = R\,C = 10 \times 10^3 \times 100 \times 10^{-6} =$ **1 s**
  - $vC(t) = 10\,(1 - e^{(-t)})$ V
  - At t = 1 s: vC = 10 (1 − 0.368) = **6.32 V**. At t = 5 s: vC = 10 (1 − 0.0067) = 9.93 V
  - Initial current i(0+) = 10 / 10 kΩ = 1 mA

➤ Second-order (series RLC) circuit

- Characteristic equation: $s^2 + (R/L)\,s + 1/(LC) = 0$.
- $\alpha = R / (2L)$ (damping factor), $\omega_0 = 1 / \sqrt{LC}$, $\zeta = \alpha / \omega_0 = (R/2)\sqrt{C/L}$.

Case	Condition	Response
Overdamped	$\zeta > 1$	Two real roots, slow, no oscillation
Critically damped	$\zeta = 1$	Repeated root, fastest without overshoot
Underdamped	$\zeta < 1$	Complex roots, decaying oscillation at $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$
Undamped	$\zeta = 0$ ($R = 0$)	Pure oscillation at ω_0

5. Sinusoidal Steady State and Resonance

- **Phasor**: $v(t) = V_m \cos(\omega t + \theta)$ is written as $V = V_m \angle\theta$. Impedance: $Z_R = R$, $Z_L = j\omega L$, $Z_C = 1/(j\omega C)$.
- **Power**: $P = V_{rms} I_{rms} \cos\phi$ (W), $Q = V_{rms} I_{rms} \sin\phi$ (VAR), $S = V_{rms} I_{rms}$ (VA). Power factor = $\cos\phi$.

FORMULA $S = P + jQ = V I^*$. $|S|^2 = P^2 + Q^2$.

➤ Series vs parallel resonance

Point	Series RLC	Parallel RLC
Resonant frequency	$f_0 = 1 / (2\pi \sqrt{LC})$	$f_0 = 1 / (2\pi \sqrt{LC})$ (ideal)
Impedance at f_0	Minimum = R	Maximum = R
Current at f_0	Maximum	Minimum (line current)
Quality factor Q	$(1/R)\sqrt{L/C} = \omega_0 L / R$	$R\sqrt{C/L} = R / (\omega_0 L)$
Also called	Acceptor circuit, voltage magnification	Rejector circuit, current magnification

FORMULA Bandwidth $BW = f_0 / Q = f_2 - f_1$. For series RLC, $BW = R / (2\pi L)$ Hz. $f_0 = \sqrt{f_1 f_2}$.

➤ Solved example (series resonance)

- $R = 10\ \Omega$, $L = 10$ mH, $C = 100$ nF.
  - $LC = 10^{-2} \times 10^{-7} = 10^{-9}$, $\sqrt{LC} = 3.162 \times 10^{-5}$ s

- $f_0 = 1 / (2\pi \times 3.162 \times 10^{-5}) =$ **5.03 kHz**
- $Q = (1/10) \times \sqrt{10^{-2} / 10^{-7}} = (1/10) \times 316.2 =$ **31.6**
- $BW = f_0 / Q = 5033 / 31.6 =$ **159 Hz** (check: $R / 2\pi L = 10 / 0.0628 = 159$ Hz)

> **TIP** In series resonance, voltage across L or C at f_0 = Q x applied voltage. That is why it is called voltage magnification.

6. Network Topology (Graph Theory)

- **Graph**: each element is a branch (line), each junction a node. **Tree**: connected subgraph with all nodes and no loops.
- **Twig** = branch in the tree; **link** (chord) = branch not in the tree.

> **FORMULA** Twigs = $n - 1$, links = $b - n + 1$. Independent KCL eqns = $n - 1$, independent KVL eqns = $b - n + 1$.

➤ Matrices

Matrix	Size	Use
Incidence matrix A (reduced)	$(n - 1) \times b$	KCL: $A\,I_b = 0$
Tie-set matrix B	$(b - n + 1) \times b$	Fundamental loops, KVL
Cut-set matrix Q	$(n - 1) \times b$	Fundamental cut-sets, KCL

- Number of possible trees = $\det(A\,A^T)$, where A is the reduced incidence matrix.
- Complete graph with n nodes has $n^{(n-2)}$ trees.

7. Laplace Transform in Circuit Analysis

- Laplace domain models: $R \to R$, $L \to sL$ (with source $L\,i(0)$ in series), $C \to 1/(sC)$ (with source $v(0)/s$ in series).
- **Initial value theorem**: $f(0+) = \lim_{s\to\infty} s\,F(s)$. **Final value theorem**: $f(\infty) = \lim_{s\to 0} s\,F(s)$ (only if poles of $sF(s)$ are in the left half plane).

➤ Network functions

- **Driving point function**: input and output at the same port (impedance $Z(s)$, admittance $Y(s)$).

- **Transfer function**: output at a different port (voltage ratio, current ratio, transfer impedance).
- Poles = roots of the denominator, zeros = roots of the numerator. For stability, poles must be in the left half of the s-plane.

EXAM POINT Driving point RC impedance: poles and zeros alternate on the negative real axis, and the pole is nearest the origin. For RL impedance, the zero is nearest the origin.

➤ Hurwitz polynomial

- All coefficients positive, no missing terms (except all-even or all-odd), roots in the LHP or on the jω axis (simple).

8. Two-Port Networks

Parameter	Equations	Used for
Z (open circuit)	$V1 = z11\ I1 + z12\ I2$, $V2 = z21\ I1 + z22\ I2$	Series connection
Y (short circuit)	$I1 = y11\ V1 + y12\ V2$, $I2 = y21\ V1 + y22\ V2$	Parallel connection
h (hybrid)	$V1 = h11\ I1 + h12\ V2$, $I2 = h21\ I1 + h22\ V2$	Transistor models, series–parallel
ABCD (transmission)	$V1 = A\ V2 - B\ I2$, $I1 = C\ V2 - D\ I2$	Cascade connection

➤ Reciprocity and symmetry conditions

Parameter	Reciprocal	Symmetrical
Z	$z12 = z21$	$z11 = z22$
Y	$y12 = y21$	$y11 = y22$
h	$h12 = -h21$	$h11\ h22 - h12\ h21 = 1$
ABCD	$AD - BC = 1$	$A = D$

- Cascade of two networks: [ABCD] total = [ABCD]1 x [ABCD]2.
- Z and Y matrices are inverses: [Y] = [Z]^-1.

EXAMPLE A series impedance Z alone has ABCD = [1 Z; 0 1]. A shunt admittance Y has ABCD = [1 0; Y 1].

9. Filters and Coupled Circuits

- **Mutual inductance**: M = k sqrt(L1 L2), coupling coefficient 0 <= k <= 1.
- Series aiding: L = L1 + L2 + 2M. Series opposing: L = L1 + L2 − 2M.
- **Dot convention**: current entering both dots → mutual voltage adds.
- Passive filters: low pass, high pass, band pass, band stop. Cutoff of RC low pass = 1 / (2πRC).
- Constant-k filter: Z1 Z2 = k^2 (independent of frequency).

10. Most Asked Questions (practice)

1. State and prove Thevenin's and Norton's theorems with an example.
2. Derive the maximum power transfer condition. Why is efficiency 50%?
3. Solve a circuit with a dependent source using nodal and mesh analysis.
4. Find the transient response of an RC and RL circuit after switching.
5. Explain overdamped, critically damped and underdamped RLC responses.
6. Derive f0, Q and bandwidth for series resonance.
7. Define incidence, tie-set and cut-set matrices for a given graph.
8. Find Z, Y, h and ABCD parameters of a T or π network.
9. State reciprocity and symmetry conditions for all two-port parameters.
10. Use Laplace transform to find the current in an RLC circuit with initial conditions.

TIP For GATE, practise Thevenin with dependent sources, first-order transients, resonance and two-port conversions. These come every year and are quick marks.