Reinforced Concrete Design

Short notes for B.Tech Civil · GATE CE

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1. Introduction and Design Philosophies

- **RCC** = concrete (good in compression) + steel bars (good in tension). They work together because of bond and nearly equal thermal expansion.
- Design code in India: **IS 456:2000** (plain and reinforced concrete). Loads from IS 875; earthquake from IS 1893.

Method	Idea	Safety
Working stress (WSM)	Stresses stay elastic under working load	Permissible stresses (factor of safety on material)
Ultimate load	Design at collapse load	Load factor
Limit state (LSM)	Safe at collapse and fit in service	Partial safety factors on loads and materials

➤ Limit states

- **Limit state of collapse**: flexure, shear, compression, torsion.
- **Limit state of serviceability**: deflection, cracking, vibration.

➤ Partial safety factors (IS 456)

Item	Factor
Concrete γm	1.5
Steel γm	1.15
Load: DL + LL	1.5
Load: DL + LL + WL / EL	1.2
Load: DL + WL (DL helps stability)	0.9 DL + 1.5 WL

- **Characteristic strength** = strength below which not more than 5% of test results fall: $f_{ck} = f_m - 1.65\,s$.
- Design strength of concrete in flexure = $0.67 f_{ck} / 1.5 = 0.446 f_{ck}$. Design strength of steel = $f_y / 1.15 = 0.87 f_y$.

FORMULA Modulus of elasticity Ec = 5000 sqrt(fck) MPa. Flexural strength fcr = 0.7 sqrt(fck) MPa. Es = 2 x 10^5 MPa.

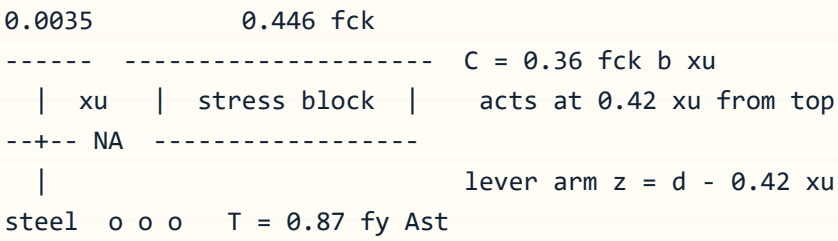
- Minimum grade for RCC: M20 (mild exposure). Nominal cover: mild 20 mm, moderate 30, severe 45, very severe 50, extreme 75 mm.

2. Limit State of Collapse: Flexure

➤ Assumptions

1. Plane sections remain plane (strain varies linearly).
2. Max strain in concrete at the outermost fibre = 0.0035.
3. Tensile strength of concrete is ignored.
4. Stress–strain of concrete: parabola up to 0.002 then constant (rectangular-parabolic).
5. Steel strain at failure ≥ 0.87 fy / Es + 0.002.

➤ Stress block



FORMULA Depth of NA: xu = 0.87 fy Ast / (0.36 fck b). Moment of resistance Mu = 0.87 fy Ast (d - 0.42 xu) = 0.36 fck b xu (d - 0.42 xu).

Steel	xu,max / d	Mu,lim
Fe 250	0.53	0.148 fck b d^2
Fe 415	0.48	0.138 fck b d^2
Fe 500	0.46	0.133 fck b d^2

- xu < xu,max: **under-reinforced** (steel yields first, ductile failure, preferred).
- xu > xu,max: **over-reinforced** (sudden concrete crushing). Not allowed; take Mu = Mu,lim.

FORMULA Ast for a given Mu: Mu = 0.87 fy Ast d [1 - (Ast fy) / (b d fck)].

EXAMPLE Beam b = 230 mm, d = 450 mm, M20, Fe415, Ast = 3 bars of 16 mm = 603 mm². xu = 0.87 x 415 x 603 / (0.36 x 20 x 230) = 217713/1656 = 131.5 mm. xu,max = 0.48 x 450 = 216 mm, so under-reinforced. Mu = 217713 x (450 - 0.42 x 131.5) = 217713 x 394.8 = 85.9 kN·m. (Mu,lim = 0.138 x 20 x 230 x 450² = 128.5 kN·m.)

➤ Steel limits in beams

- Minimum tension steel: As / (b d) = 0.85 / fy.
- Maximum tension or compression steel: 0.04 b D each.
- Side face reinforcement when depth of web > 750 mm.

3. Doubly Reinforced and Flanged Beams

- **Doubly reinforced** when Mu > Mu,lim and depth is restricted. Extra moment (Mu − Mu,lim) is taken by compression steel + extra tension steel.

FORMULA Mu - Mu,lim = fsc Asc (d - d'). Extra tension steel Ast2 = fsc Asc / (0.87 fy). fsc from strain at level d'.

➤ T-beams and L-beams

FORMULA Effective flange width: T-beam bf = l0/6 + bw + 6 Df. L-beam bf = l0/12 + bw + 3 Df. (l0 = distance between points of zero moment; bf not more than actual width.)

- If xu ≤ Df: NA in flange, design as rectangular beam of width bf.
- If xu > Df: NA in web; flange and web parts are taken separately.

4. Shear, Bond and Torsion

FORMULA Nominal shear stress τv = Vu / (b d). If τv > τc,max, revise the section.

- τc (design shear strength of concrete) depends on fck and % tension steel pt (IS 456 Table 19).

- $\tau_{c,max}$ (Table 20): M20 = 2.8 MPa, M25 = 3.1 MPa, M30 = 3.5 MPa.

Case	Action
$\tau_v < 0.5\,\tau_c$	No stirrups needed (except in beams, where minimum is always given)
$0.5\,\tau_c < \tau_v < \tau_c$	Minimum shear reinforcement
$\tau_c < \tau_v < \tau_{c,max}$	Design shear reinforcement for $V_{us} = V_u - \tau_c\, b\, d$

FORMULA Vertical stirrups: $V_{us} = 0.87\, f_y\, A_{sv}\, d / s_v$. Minimum stirrups: $A_{sv} / (b\, s_v) \geq 0.4 / (0.87\, f_y)$. Max spacing = 0.75 d or 300 mm (smaller).

EXAMPLE V_u = 150 kN, b = 230, d = 450, M20, Fe415, τ_c = 0.48 MPa (pt about 0.5%). τ_v = 150000/(230 x 450) = 1.45 MPa < 2.8 OK. V_{us} = 150 - 0.48 x 230 x 450/1000 = 150 - 49.7 = 100.3 kN. With 2-legged 8 mm stirrups, A_{sv} = 100.5 mm². s_v = 0.87 x 415 x 100.5 x 450 / 100300 = 162.8 mm. Provide 8 mm 2-legged stirrups at 160 mm c/c (< 0.75d = 337 mm and < 300 mm).

➤ Bond and development length

FORMULA $L_d = \phi\, \sigma_s / (4\, \tau_{bd})$, $\sigma_s = 0.87\, f_y$. For deformed bars τ_{bd} is increased by 60%. M20 with Fe415 (deformed): $L_d \approx 47\, \phi$.

- τ_{bd} for plain bars in tension, M20 = 1.2 MPa (deformed bars: 1.92 MPa). In compression, increase by 25%.

➤ Torsion (IS 456)

FORMULA Equivalent shear $V_e = V_u + 1.6\, T_u / b$. Equivalent moment $M_{el} = M_u + M_t$, where $M_t = T_u\,(1 + D/b) / 1.7$.

5. Slabs

- **One-way slab**: $l_y / l_x > 2$ (bends in short direction). **Two-way slab**: $l_y / l_x \leq 2$.
- Two-way slab moments: $M_x = \alpha_x\, w\, l_x^2$, $M_y = \alpha_y\, w\, l_x^2$ (coefficients from IS 456 Annex D). Corners of restrained slabs need torsion steel.

Rule (slabs)	Value
Minimum steel	0.12% of bD (HYSD), 0.15% (mild steel)
Max bar diameter	D / 8
Max spacing of main bars	3d or 300 mm (smaller)
Max spacing of distribution bars	5d or 450 mm (smaller)

➤ Deflection control (span / effective depth)

Support	Basic l/d (span up to 10 m)
Cantilever	7
Simply supported	20
Continuous	26

- Modify by factors for tension steel (stress level), compression steel and flanges.
- Other types: flat slab (no beams, drops and column heads), waffle slab, ribbed slab.

6. Columns

- **Short column**: lex / D and ley / b both < 12. Otherwise **long (slender) column**: add moments due to buckling.
- **Minimum eccentricity** emin = l / 500 + D / 30, not less than 20 mm.

FORMULA Short axially loaded column (when emin ≤ 0.05 D): Pu = 0.4 fck Ac + 0.67 fy Asc. With helical ties: strength x 1.05.

EXAMPLE Column 400 x 400 mm, M25, Fe415, Asc = 1% = 1600 mm². Ac = 160000 - 1600 = 158400 mm². Pu = 0.4 x 25 x 158400 + 0.67 x 415 x 1600 = 1584000 + 444880 = 2028880 N = 2029 kN.

Column detailing rule	Value
Longitudinal steel	0.8% to 4% of gross area (6% absolute max)
Min number of bars	4 (rectangular), 6 (circular)
Min bar diameter	12 mm
Tie diameter	≥ ϕ/4 and ≥ 6 mm
Tie pitch	≤ least lateral dimension, 16 ϕ, 300 mm

- Columns with axial load + moment: use **interaction diagrams** (SP 16). Biaxial bending: (Mux/Mux1)^αn + (Muy/Muy1)^αn ≤ 1.

7. Footings, Stairs and Other Elements

- Footing design checks: area from safe bearing capacity (service loads), bending at face of column, one-way shear at d from face, two-way (punching) shear at d/2 from face, bearing, development length.

FORMULA Punching shear: τv ≤ ks τc, τc = 0.25 sqrt(fck), ks = 0.5 + βc ≤ 1 (βc = short side / long side of column).

- Minimum depth at edge of footing on soil: 150 mm. Nominal cover for footings: at least 50 mm.
- **Stairs**: waist slab design as one-way slab; effective span depends on supports and landings.
- **Retaining walls**: stem, toe and heel designed as cantilevers.
- **Working stress method** basics: modular ratio m = 280 / (3 σcbc).

REMEMBER Ductile detailing for earthquakes follows IS 13920 (closely spaced stirrups near joints, 135° hooks).

8. Most Asked Questions (practice)

1. Explain the limit state method and partial safety factors in IS 456.
2. State the assumptions of limit state of flexure; derive xu,max / d for Fe415.
3. Find Mu of a singly reinforced beam; check under / over reinforcement.
4. Design a doubly reinforced beam for a given moment.
5. Find the effective flange width and Mu of a T-beam.
6. Design shear reinforcement for a beam.
7. Find development length for given bar and grade.
8. Design a one-way and a two-way slab with deflection check.
9. Design a short axially loaded column with ties.
10. Design an isolated square footing with one-way and punching shear checks.

TIP GATE CE asks xu,max, Mu,lim, Pu of columns, shear checks, Ld, flange width and safety factors. Remember the IS 456 numbers exactly; they give easy marks.