Soil Mechanics

Short notes for B.Tech Civil · GATE CE

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I. Soil Formation and Phase Relations

- **Soil** = loose material formed by weathering of rocks. **Residual soil** stays in place; **transported soil** is moved by water (alluvial), wind (aeolian, loess), ice (glacial) or gravity (colluvial).
- Soil has three phases: solids, water, air. Dry soil = 2 phases (solids + air); saturated soil = 2 phases (solids + water).

➤ Basic terms

Term	Formula
Void ratio e	Vv / Vs
Porosity n	Vv / V (n = e / (1 + e))
Degree of saturation S	Vw / Vv
Water content w	Ww / Ws
Air content ac	Va / Vv
Percentage air voids na	Va / V = n ac

FORMULA Se = w G. Dry unit weight $\gamma d = G \gamma w / (1 + e) = \gamma / (1 + w)$.

FORMULA Bulk $\gamma = (G + S e) \gamma w / (1 + e)$. Saturated $\gamma sat = (G + e) \gamma w / (1 + e)$. Submerged $\gamma' = \gamma sat - \gamma w = (G - 1) \gamma w / (1 + e)$.

EXAMPLE w = 20%, G = 2.7, e = 0.6. S = wG/e = 0.54/0.6 = 90%. γd = 2.7 x 9.81/1.6 = 16.55 kN/m³. γ = 16.55 x 1.2 = 19.86 kN/m³. γsat = 3.3 x 9.81/1.6 = 20.23 kN/m³.

- **Relative density** Dr = (emax − e) / (emax − emin). Used for sands. Dr < 15% very loose, > 85% very dense.

2. Index Properties and Classification

➤ Particle size

- Sieve analysis for coarse soil (> 75 μm), hydrometer / sedimentation (Stokes' law) for fine soil.
- IS sizes: gravel 4.75 to 80 mm, sand 0.075 to 4.75 mm, silt 0.002 to 0.075 mm, clay < 0.002 mm.

FORMULA Uniformity coefficient $Cu = D60 / D10$. Curvature coefficient $Cc = D30^2 / (D10 \times D60)$.

- Well graded: Cc between 1 and 3, and Cu > 4 (gravel) or Cu > 6 (sand). Cu = 1 means uniform (poorly graded).
- D10 = **effective size** (used in Hazen's formula $k = C\ D10^2$).

➤ Atterberg limits (consistency)

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solid | semi-solid | plastic | liquid  --> water content
      SL           PL        LL
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- **Liquid limit (LL)**: Casagrande cup, water content at 25 blows to close a 13 mm groove (or cone penetration 20 mm).
- **Plastic limit (PL)**: soil thread crumbles at 3 mm diameter.
- **Shrinkage limit (SL)**: below this, no more volume decrease on drying.

FORMULA Plasticity index $PI = LL - PL$. Liquidity index $LI = (w - PL)/PI$. Consistency index $CI = (LL - w)/PI$. Activity $A = PI$ / (% clay finer than 2 μm).

- Activity: < 0.75 inactive (kaolinite), 0.75 to 1.25 normal (illite), > 1.25 active (montmorillonite, black cotton soil).
- **Sensitivity** St = unconfined strength undisturbed / remoulded. **Thixotropy** = strength regain with time after remoulding.

➤ IS classification (IS 1498)

- Coarse grained: more than 50% retained on 75 μm sieve (G or S). Fine grained: more than 50% passes 75 μm (M, C, O).
- Fine soils: L (LL < 35), I (35 to 50), H (LL > 50).

- **Plasticity chart A-line**: PI = 0.73 (LL − 20). Above A-line = clay (C), below = silt (M) or organic (O).

3. Permeability and Seepage

FORMULA Darcy's law v = k i, Q = k i A. Seepage velocity vs = v / n.

- Constant head test (coarse soils): k = Q L / (A h t).
- Falling head test (fine soils): k = 2.303 (a L / A t) log10(h1/h2).
- k depends on grain size, void ratio, viscosity (temperature) and soil structure.

➤ Layered soil

FORMULA Horizontal flow kh = Σ(ki Hi) / ΣHi. Vertical flow kv = ΣHi / Σ(Hi / ki). Always kh > kv.

➤ Seepage and flow nets

- **Laplace equation** governs 2D steady seepage: $\partial^2h/\partial x^2 + \partial^2h/\partial z^2 = 0$.
- Flow net = flow lines + equipotential lines forming curvilinear squares.

FORMULA Seepage per metre width q = k H Nf / Nd (Nf = flow channels, Nd = potential drops).

- **Quick sand**: effective stress becomes zero due to upward flow. Critical gradient ic = (G − 1) / (1 + e), about 1.0.
- Remedy for piping: longer seepage path, filters, cut-off walls, loading at exit.

4. Effective Stress and Compaction

- **Terzaghi's principle**: $\sigma' = \sigma - u$. Strength and settlement depend only on effective stress.
- Upward seepage reduces σ'; downward seepage increases σ'.
- **Capillary zone**: water held by surface tension above the water table; pore pressure is negative there, so σ' increases.

➤ Compaction

- Compaction = removing air by mechanical energy (quick). Consolidation = removing water under sustained load (slow).
- **OMC** = water content giving the maximum dry density (MDD).

Test	Rammer	Drop	Layers	Blows per layer
Standard Proctor	2.6 kg	310 mm	3	25
Modified Proctor	4.89 kg	450 mm	5	25

- More compaction energy → higher MDD, lower OMC.
- Dry of optimum: flocculated structure, higher strength, more permeable. Wet of optimum: dispersed structure, lower permeability, more flexible (used in dam cores).

FORMULA Zero air voids line: $\gamma d = G \gamma w / (1 + w G)$.

- Field control: sand replacement method, core cutter method. Rollers: smooth wheel (granular base), sheep-foot (clay), vibratory (sand), pneumatic (mixed).

5. Consolidation

- **Primary consolidation**: volume change due to expulsion of pore water as excess pore pressure dissipates.
- **Secondary compression**: creep after excess pore pressure is zero (important in organic soils).
- **Normally consolidated (NC)**: present σ' is the maximum ever. **Over consolidated (OC)**: past pressure was higher. OCR = $\sigma'c / \sigma'0$.
- Preconsolidation pressure $\sigma'c$ found by Casagrande's construction on the e–log σ' curve.

Parameter	Formula
Coefficient of compressibility av	$-\Delta e / \Delta \sigma'$
Coefficient of volume change mv	$av / (1 + e0)$
Compression index Cc	$\Delta e / \log10(\sigma'2/\sigma'1)$
Coefficient of consolidation cv	$k / (mv \gamma w)$

FORMULA Settlement of NC clay: $Sc = Cc\ H / (1 + e0) \times \log10[(\sigma'0 + \Delta\sigma) / \sigma'0]$. Also $Sc = mv\ \Delta\sigma\ H$.

FORMULA Time factor $T_v = c_v t / d^2$. d = drainage path ($H/2$ for double drainage, H for single). $T_v = 0.197$ at $U = 50\%$, $T_v = 0.848$ at $U = 90\%$.

- For $U < 60\%$: $T_v = (\pi/4) U^2$. Skempton: $C_c = 0.009 (LL - 10)$ for undisturbed clay.

EXAMPLE NC clay layer $H = 4$ m, $e_0 = 0.9$, $C_c = 0.3$, $\sigma'_0 = 100$ kPa, $\Delta\sigma = 50$ kPa. $S_c = 0.3 \times 4/1.9 \times \log10(150/100) = 0.6316 \times 0.1761 = 0.111$ m = 111 mm.

TIP Time for a given U is proportional to d^2. Double drainage makes consolidation 4 times faster than single drainage for the same layer.

6. Shear Strength

FORMULA Mohr-Coulomb: $\tau_f = c' + \sigma' \tan \phi'$. Failure plane makes $(45° + \phi/2)$ with the major principal plane.

FORMULA $\sigma_1 = \sigma_3 \tan^2(45° + \phi/2) + 2c \tan(45° + \phi/2)$.

Test	Drainage	Use
UU (quick)	No drainage in both stages	End of construction in clay
CU	Consolidated, undrained shear (pore pressure measured)	Rapid drawdown, staged loading
CD (slow)	Drained in both stages	Long-term stability
Unconfined compression	$\sigma_3 = 0$, $c_u = q_u/2$	Saturated clay
Vane shear	In-situ, undrained	Soft sensitive clay
Direct shear	Failure plane fixed	Sand, quick and simple

- For saturated clay in UU test, $\phi_u = 0$ and failure envelope is horizontal.
- Dense sand dilates (volume increase) and shows peak strength; loose sand contracts.
- **Skempton's pore pressure parameters**: $\Delta u = B [\Delta\sigma_3 + A (\Delta\sigma_1 - \Delta\sigma_3)]$. $B = 1$ for saturated soil.

7. Stress Distribution in Soil

FORMULA Boussinesq point load: $\sigma z = (3Q / 2\pi z^2) \times [1 / (1 + (r/z)^2)]^{(5/2)}$. Directly below the load ($r = 0$): $\sigma z = 0.4775\ Q / z^2$.

- **Westergaard** solution is for layered (anisotropic) soils; gives lower stresses.
- **2:1 method**: $\Delta\sigma = Q / [(B + z)(L + z)]$ for a rectangular footing. Quick approximate method.
- **Pressure bulb**: zone inside a stress contour (isobar). A strip footing's bulb goes much deeper than a square footing's bulb of the same width.
- Newmark's influence chart: $\Delta\sigma = q \times$ number of blocks $\times$ influence value (0.005 for 200-block chart).

8. Earth Pressure and Slope Stability

State	Coefficient	Wall movement
At rest K0	$1 - \sin\phi$ (Jaky)	None
Active Ka	$(1 - \sin\phi) / (1 + \sin\phi)$	Away from soil
Passive Kp	$(1 + \sin\phi) / (1 - \sin\phi) = 1/Ka$	Towards soil

- Always $Ka < K0 < Kp$. Passive state needs much larger movement than active.

FORMULA Active thrust (dry, $c = 0$): $Pa = 0.5\ Ka\ \gamma\ H^2$, acting at $H/3$ above the base. Cohesive soil tension crack depth $zc = 2c / (\gamma\ \mathrm{sqrt}(Ka))$.

- Surcharge q adds $Ka\ q$ uniformly. Water table: use γ' for soil and add full water pressure $\gamma w\ h$.
- **Coulomb's wedge theory** includes wall friction δ and sloping backfill; Rankine assumes a smooth wall.

EXAMPLE Wall $H = 6$ m, dry sand $\gamma = 18$ kN/m³, $\phi = 30°$. $Ka = (1 - 0.5)/(1 + 0.5) = 1/3$. $Pa = 0.5 \times (1/3) \times 18 \times 6^2 = 108$ kN/m, acting 2 m above the base.

Slope stability

- Infinite slope, dry cohesionless soil: $FS = \tan\phi / \tan\beta$. Submerged: same formula.

- Infinite slope with seepage parallel to slope: $FS = (\gamma' / \gamma sat) \tan \phi / \tan \beta$ (about half).
- Finite slopes: Swedish slip circle ($\phi = 0$), method of slices (Fellenius, Bishop), Taylor's stability number $Sn = c / (F \gamma H)$.

9. Most Asked Questions (practice)

1. Derive $\gamma d = G\gamma w/(1 + e)$ and $Se = wG$. Solve a three-phase numerical.
2. Explain Atterberg limits and the IS plasticity chart.
3. Derive the falling head permeability formula.
4. Find equivalent permeability of layered soil in both directions.
5. What is quick sand? Derive the critical hydraulic gradient.
6. Explain effective stress with and without seepage; draw stress diagrams.
7. Differentiate compaction and consolidation. Explain the Proctor test.
8. Explain Terzaghi's 1D consolidation theory; find settlement and time.
9. Explain UU, CU and CD tests with their field uses.
10. Find active and passive thrust on a retaining wall with water table and surcharge.

TIP GATE CE asks phase relations, flow net seepage, consolidation time, effective stress and Rankine pressure every year. Draw neat stress diagrams before you calculate.