Strength of Materials

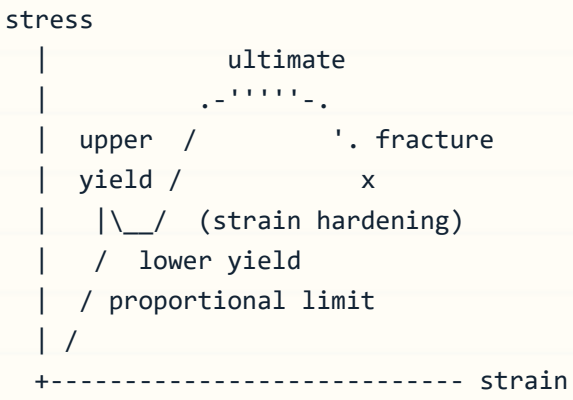
Short notes for B.Tech Civil · GATE CE

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1. Simple Stress and Strain

- **Stress** = internal resistance per unit area. $\sigma = P / A$. Unit: $N/mm^2 = MPa$.
- **Strain** = change in length / original length. $\varepsilon = \delta L / L$. No unit.
- **Hooke's law**: within the elastic limit, stress is proportional to strain. $\sigma = E\,\varepsilon$.
- **Poisson's ratio** μ = lateral strain / longitudinal strain. For steel about 0.3, for concrete about 0.15 to 0.2.
- μ lies between 0 and 0.5. $\mu = 0.5$ means the material is incompressible (no volume change).

➤ Stress–strain curve of mild steel



- Order of points: proportional limit, elastic limit, upper yield, lower yield, ultimate, fracture.
- **Ductile** materials (mild steel) show large plastic strain. **Brittle** materials (cast iron, concrete) fail with little strain.

FORMULA Elongation of a bar: $\delta = PL / (AE)$. Bar of varying sections: $\delta = (P/E) \times \Sigma(Li / Ai)$.

FORMULA Tapering circular bar (d1 to d2): $\delta = 4PL / (\pi\, E\, d1\, d2)$. Bar hanging under own weight: $\delta = \gamma L^2 / (2E)$.

Elastic constants

Constant	Definition
E (Young's modulus)	normal stress / normal strain
G (Rigidity modulus)	shear stress / shear strain
K (Bulk modulus)	direct stress / volumetric strain
μ (Poisson's ratio)	lateral strain / linear strain

FORMULA $E = 2G(1 + \mu) = 3K(1 - 2\mu)$. Combined: $E = 9KG / (3K + G)$.

- Volumetric strain of a bar: $\varepsilon v = \varepsilon(1 - 2\mu)$. For a sphere: $\varepsilon v = 3 \times$ diametral strain. For a cylinder: $\varepsilon v = 2\varepsilon d + \varepsilon L$.

Thermal stress and composite bars

- Free bar: expands by $\delta = \alpha L \Delta T$, no stress.
- Fully restrained bar: $\sigma = E \alpha \Delta T$ (compression on heating).
- **Composite bar** (steel rod inside copper tube): same strain in both, load shared: $P = \sigma s\, As + \sigma c\, Ac$.

EXAMPLE Steel bar $d = 20$ mm, $L = 2$ m, $P = 50$ kN, $E = 200$ GPa. $A = \pi/4 \times 20^2 = 314.16$ mm². $\sigma = 50000/314.16 = 159.2$ MPa. $\delta = PL/AE = 50000 \times 2000 / (314.16 \times 200000) = 1.59$ mm.

Strain energy

- **Strain energy** U = work stored in the body $= 1/2 \times P \times \delta = \sigma^2 \times V / (2E)$.
- **Resilience** = strain energy per unit volume $= \sigma^2/(2E)$. **Proof resilience** = value at elastic limit.

Loading type	Stress produced
Gradual load P	$\sigma = P/A$
Sudden load P	$\sigma = 2P/A$ (double)
Impact (fall h)	$\sigma = (P/A)[1 + \text{sqrt}(1 + 2AEh/(PL))]$

2. Shear Force and Bending Moment

- **Shear force (SF)** at a section = algebraic sum of vertical forces on one side.

- **Bending moment (BM)** at a section = algebraic sum of moments of forces on one side.
- Sign: sagging BM positive, hogging BM negative.

➤ Key relations

- $dM/dx = V$ (slope of BM diagram = SF). $dV/dx = -w$ (slope of SF diagram = load intensity).
- BM is maximum where SF is zero or changes sign.
- **Point of contraflexure**: where BM changes sign (only in overhanging and fixed / continuous beams).

Load on beam	SFD shape	BMD shape
Point load	Rectangle (step)	Triangle (linear)
UDL	Triangle (linear)	Parabola (2nd degree)
UVL (triangular)	Parabola	Cubic
Couple	No change	Sudden jump

FORMULA SSB, central point load W: $M_{max} = WL/4$. SSB, UDL w: $M_{max} = wL^2/8$. Cantilever, end load W: $M_{max} = WL$. Cantilever, UDL w: $M_{max} = wL^2/2$.

3. Bending and Shear Stresses in Beams

- Assumptions: plane sections remain plane, material is homogeneous and obeys Hooke's law, same E in tension and compression.
- **Neutral axis** passes through the centroid; bending stress is zero there and maximum at extreme fibres.

FORMULA Flexure formula: $M / I = \sigma / y = E / R$. Section modulus $Z = I / y_{max}$, so $\sigma_{max} = M / Z$.

Section	I about NA	Z
Rectangle b x d	$bd^3/12$	$bd^2/6$
Solid circle d	$\pi d^4/64$	$\pi d^3/32$
Hollow circle D, d	$\pi(D^4 - d^4)/64$	$\pi(D^4 - d^4)/(32D)$
Triangle b x h (about centroid)	$bh^3/36$	—

- **Parallel axis theorem**: $I = I_G + A h^2$. **Perpendicular axis theorem**: $I_z = I_x + I_y$ (plane areas).

➤ Shear stress distribution

- $\tau = V A \bar{y} / (I b)$, where $A \bar{y}$ = first moment of area above the level, b = width at that level.
- Rectangle: parabolic, $\tau_{max} = 1.5 \times \tau_{avg}$ at NA.
- Circle: $\tau_{max} = (4/3) \times \tau_{avg}$ at NA.
- Triangle: $\tau_{max} = 1.5\ \tau_{avg}$ at mid-height (not at NA); at NA it is $(4/3)\ \tau_{avg}$.
- I-section: most of the shear is taken by the web; a jump occurs at the flange–web junction.

EXAMPLE SSB span 6 m, UDL 10 kN/m, section 200 x 300 mm. $M_{max} = 10 \times 6^2/8 = 45$ kN·m. $Z = 200 \times 300^2/6 = 3 \times 10^6$ mm³. $\sigma_{max} = 45 \times 10^6 / 3 \times 10^6 = 15$ MPa. $V_{max} = 10 \times 6/2 = 30$ kN. $\tau_{avg} = 30000/60000 = 0.5$ MPa, $\tau_{max} = 1.5 \times 0.5 = 0.75$ MPa.

4. Deflection of Beams

- Differential equation of elastic curve: $EI\, d^2y/dx^2 = M$.
- Methods: double integration, Macaulay's method, moment-area method, conjugate beam method, strain energy (Castigliano).
- **Moment-area theorem 1**: change in slope between two points = area of M/EI diagram between them.
- **Moment-area theorem 2**: deflection of B from tangent at A = moment of M/EI area about B.

Beam and load	Max slope	Max deflection
Cantilever, end load W	$WL^2/(2EI)$	$WL^3/(3EI)$
Cantilever, UDL w	$wL^3/(6EI)$	$wL^4/(8EI)$
SSB, central load W	$WL^2/(16EI)$	$WL^3/(48EI)$
SSB, UDL w	$wL^3/(24EI)$	$5wL^4/(384EI)$
Fixed beam, central load W	0 at ends	$WL^3/(192EI)$
Fixed beam, UDL w	0 at ends	$wL^4/(384EI)$

REMEMBER Fixed beam with UDL deflects 1/5 of the simply supported beam ($wL^4/384$ vs $5wL^4/384$).

TIP In conjugate beam method, a fixed end becomes free and a free end becomes fixed; hinges at supports stay hinges.

5. Torsion of Shafts

- Assumptions: circular sections remain plane, radii stay straight, material is elastic and homogeneous.

FORMULA Torsion equation: $T / J = \tau / r = G\,\theta / L$. Solid shaft: $J = \pi d^4/32$, $\tau_{max} = 16T/(\pi d^3)$. Hollow shaft: $J = \pi(D^4 - d^4)/32$.

FORMULA Power transmitted $P = 2\pi NT / 60$ (P in W, N in rpm, T in N·m).

- **Torsional stiffness** = $T/\theta = GJ/L$. **Polar section modulus** $Zp = J / R$.
- Hollow shaft is stronger and stiffer than a solid shaft of the same weight.
- Shafts in series: same torque, angles add. Shafts in parallel: same angle, torques add.

EXAMPLE Solid shaft $d = 100$ mm, $N = 200$ rpm, $T = 10$ kN·m. $\tau_{max} = 16 \times 10^7 / (\pi \times 100^3) = 50.9$ MPa. Power $= 2\pi \times 200 \times 10 / 60 = 209.4$ kW.

➤ Close-coiled helical spring

- Wire diameter d, mean coil radius R, n turns, axial load W.

FORMULA $\tau = 16WR / (\pi d^3)$. Deflection $\delta = 64\,W\,R^3\,n / (G\,d^4)$. Stiffness $k = W / \delta = G\,d^4 / (64\,R^3\,n)$.

- Springs in series: $1/k = 1/k1 + 1/k2$. Springs in parallel: $k = k1 + k2$.

6. Principal Stresses and Mohr's Circle

- **Principal planes** = planes with zero shear stress. Normal stresses on them = **principal stresses**.
- Principal planes are 90° apart; planes of max shear are at 45° to them.

FORMULA $\sigma_{1,2} = (\sigma_x + \sigma_y)/2 \pm \sqrt{[((\sigma_x - \sigma_y)/2)^2 + \tau_{xy}^2]}$. $\tan 2\theta_p = 2\tau_{xy} / (\sigma_x - \sigma_y)$. $\tau_{max} = (\sigma_1 - \sigma_2)/2$.

➤ Mohr's circle

- Centre at $((\sigma_x + \sigma_y)/2, 0)$, radius = $\sqrt{[((\sigma_x - \sigma_y)/2)^2 + \tau_{xy}^2]}$.
- Radius of the circle = maximum in-plane shear stress.
- Angle 2θ on the circle = angle θ on the element.

EXAMPLE $\sigma_x = 80$ MPa, $\sigma_y = 20$ MPa, $\tau_{xy} = 40$ MPa. Centre = 50, radius = $\sqrt{(30^2 + 40^2)} = 50$. $\sigma_1 = 100$ MPa, $\sigma_2 = 0$, $\tau_{max} = 50$ MPa. $\tan 2\theta = 80/60$, $2\theta = 53.13°$, $\theta = 26.57°$.

➤ Theories of failure

Theory	Failure when	Suitable for
Max principal stress (Rankine)	$\sigma_1 = f_y$	Brittle materials
Max shear stress (Tresca / Guest)	$\sigma_1 - \sigma_2 = f_y$	Ductile, safe side
Max principal strain (St. Venant)	$\sigma_1 - \mu\sigma_2 = f_y$	Rarely used
Total strain energy (Haigh)	$\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2 = f_y^2$	Ductile
Max distortion energy (von Mises)	$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = f_y^2$	Best for ductile

EXAM POINT Pure shear: Tresca gives $\tau_y = 0.5\ f_y$, von Mises gives $\tau_y = 0.577\ f_y$ $(= f_y/\sqrt{3})$.

7. Columns and Struts

- **Short column** fails by crushing, **long column** by buckling.
- **Slenderness ratio** $\lambda = L_e / r_{min}$, where $r = \sqrt{(I/A)}$.

FORMULA Euler's crippling load $P_{cr} = \pi^2 E I_{min} / L_e^2$. Valid only for long columns (λ above a limit, about 80 to 100 for mild steel).

End condition	Effective length Le	Pcr compared to hinged
Both ends hinged	L	1
Both ends fixed	$L/2$	4 times
One fixed, one hinged	$L/\sqrt{2}$	2 times
One fixed, one free	$2L$	1/4

- **Rankine's formula** (all lengths): $1/P = 1/P_c + 1/P_E$, i.e. $P = \sigma_c A / (1 + a\lambda^2)$.

8. Thin Cylinders, Springs and Combined Stresses

- Thin cylinder: wall thickness $t < d/20$ (about). Stress is taken as uniform across the wall.

FORMULA Hoop stress $\sigma_h = pd / (2t)$. Longitudinal stress $\sigma_l = pd / (4t)$. Thin sphere: $\sigma = pd / (4t)$.

- Hoop stress is twice the longitudinal stress, so a cylinder fails along its length (longitudinal joint is critical).
- Thick cylinders: **Lamé's equations** $\sigma_r = A - B/r^2$, $\sigma_h = A + B/r^2$; max hoop stress at the inner surface.
- **Combined bending and direct stress**: $\sigma = P/A \pm M/Z$. No tension condition for rectangle: $e \le b/6$ (**middle third rule**). For circle: $e \le d/8$ (**core diameter d/4**).

REMEMBER Kernel (core) of a rectangular section is a rhombus with diagonals $b/3$ and $d/3$.

9. Most Asked Questions (practice)

1. Derive the relation $E = 2G(1 + \mu)$. Find K when $E = 200$ GPa and $\mu = 0.3$.
2. Draw SFD and BMD for an overhanging beam and locate the point of contraflexure.
3. Derive the flexure formula $M/I = \sigma/y = E/R$. State the assumptions.
4. Show that $\tau_{max} = 1.5\,\tau_{avg}$ for a rectangular section.
5. Find the slope and deflection of a SSB with UDL using Macaulay's method.
6. Derive the torsion equation. Compare a hollow and solid shaft of equal weight.
7. Find principal stresses and their planes using Mohr's circle.
8. Derive Euler's formula for a column hinged at both ends. State its limitations.
9. Find the stress due to impact loading on a bar.
10. Find hoop and longitudinal stress in a thin cylinder and the change in its volume.

TIP For GATE CE, practise Mohr's circle, deflection formulas, Euler effective lengths, elastic constants and shear stress distribution. Learn the standard formula tables by heart.