Theory of Machines

Short notes for B.Tech Mechanical · GATE ME

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1. Mechanisms and Machines

- **Kinematic link**: a resistant body that is part of a machine and has relative motion (e.g. crank, connecting rod).
- **Kinematic pair**: two links in contact with constrained relative motion.
- **Kinematic chain**: links joined so that relative motion is fully constrained. **Mechanism**: a chain with one link fixed. **Machine**: a mechanism that transmits force or does useful work.

➤ Types of pairs

Basis	Types
Contact	Lower pair (surface contact), higher pair (point / line contact, e.g. cam, gears)
Motion	Turning, sliding, rolling, screw, spherical
Closure	Self-closed, force-closed (spring or gravity keeps contact)

FORMULA Grubler / Kutzbach (planar): $F = 3(n - 1) - 2j - h$ (n = links, j = lower pairs, h = higher pairs).

- $F = 1$: constrained mechanism. $F = 0$: structure. $F < 0$: redundant (super) structure.
- Four-bar chain: $n = 4$, $j = 4$, so $F = 3 \times 3 - 8 = 1$.

➤ Grashof's law (four-bar)

- If $s + l \leq p + q$ (shortest + longest <= sum of other two), at least one link can rotate fully.
  - Shortest link fixed: **double crank** (drag link). Link next to the shortest fixed: **crank–rocker**. Link opposite the shortest fixed: **double rocker**.

➤ Inversions

Chain	Inversion examples
Four-bar	Beam engine, coupled wheels of locomotive, Watt's indicator
Single slider crank	Reciprocating engine, Whitworth quick return, rotary engine, oscillating cylinder engine, crank and slotted lever, hand pump
Double slider crank	Elliptical trammel, Scotch yoke, Oldham coupling

TIP Quick return ratio = time of cutting stroke / time of return stroke = α / β, where α and β are the crank angles for the cutting and return strokes (α > β, α + β = 360°).

2. Velocity and Acceleration Analysis

- **Instantaneous centre (IC)**: point where two links have no relative velocity at that instant.

FORMULA Number of ICs N = n(n − 1)/2. Kennedy's theorem: the three ICs of any three links lie on one straight line.

- Relative velocity method: vBA = ω x AB, perpendicular to AB.
- Acceleration of B relative to A has two parts: **centripetal** ω^2 x AB (towards A) and **tangential** α x AB (normal to AB).
- **Coriolis component**: appears when a slider moves along a rotating link.

FORMULA Coriolis acceleration = 2 v ω (v = sliding velocity, ω = angular velocity of the link).

- Slider-crank (approx.): piston acceleration ≈ ω^2 r (cosθ + cos2θ / n), n = l/r.

3. Cams and Followers

- **Cam** = a link that gives a desired motion to a **follower** by direct contact.
- Followers: knife edge, roller, flat-faced, spherical. Motion: reciprocating or oscillating; in-line or offset.

- Terms: base circle (smallest circle on cam profile), pitch curve, prime circle, pressure angle (angle between follower motion and the common normal), lift (stroke).

Follower motion	Velocity	Acceleration	Note
Uniform velocity	Constant	Infinite at ends	Only low speed
SHM	Sine	Sudden jump at ends	Medium speed
Uniform acc. and retardation	Linear	Constant	Lowest max acceleration
Cycloidal	Smooth	Smooth, zero at ends	Best for high speed (no jerk jump)

- Keep pressure angle small (usually below 30°) to reduce side thrust; increase base circle size to reduce it.

4. Gears

- **Law of gearing**: the common normal at the point of contact must always pass through a fixed **pitch point**. This gives a constant velocity ratio.

FORMULA Module $m = d / T$ (mm). Circular pitch $p = \pi m$. Diametral pitch $P = T / d$. Velocity ratio $N1/N2 = T2/T1 = d2/d1$.

Point	Involute teeth	Cycloidal teeth
Pressure angle	Constant	Varies
Centre distance change	Velocity ratio unchanged	Must be exact
Interference	Can occur	Does not occur
Manufacture	Easy (one rack cutter)	Harder
Use	Almost all power gears	Clocks, instruments

- Standard pressure angles: 14.5°, 20° (most common), 25°.
- **Interference**: tip of one tooth digs into the non-involute flank of the other. Avoid by more teeth, larger pressure angle, stub teeth or undercutting.
- Min teeth on a pinion meshing with a rack to avoid interference: about 17 for 20° full depth, 32 for 14.5°.
- **Contact ratio** = arc of contact / circular pitch (should be > 1, usually 1.2 to 1.6).

- Gear types: spur and helical (parallel shafts), bevel (intersecting), worm and worm wheel (non-intersecting, high ratio, can be self-locking).

5. Gear Trains

Train	Idea	Speed ratio
Simple	One gear per shaft	N1/Nn = Tn/T1 (idlers do not change ratio, only direction)
Compound	Two gears on a shaft	Product of driven teeth / product of driving teeth
Reverted	First and last gear coaxial	Clocks, lathe back gear
Epicyclic	Axis of a gear moves (planet)	Use tabular method

- **Epicyclic (tabular) method**: 1) fix the arm, give the sun +1 turn; 2) multiply by x; 3) add y to all (arm turns y). Use known speeds to find x and y.
- Torque balance in an epicyclic train: T_input + T_output + T_fixing = 0.
- Used in automatic gearboxes, differentials and hub gears.

6. Belts, Ropes and Chains

FORMULA Flat belt: T1/T2 = e^(μθ). V-belt or rope: T1/T2 = e^(μθ / sinβ) (β = half groove angle).

FORMULA Power P = (T1 − T2) v. Centrifugal tension Tc = m v^2 (m = mass per metre).

- For max power: T_max = 3 Tc, so v = sqrt(T_max / (3m)).
- Velocity ratio N2/N1 = d1/d2 (with belt thickness: (d1 + t)/(d2 + t)). **Slip** reduces it by (1 − s/100).
- **Creep**: small relative motion due to elastic stretch difference between tight and slack sides.
- Open belt length L = π(r1 + r2) + 2x + (r1 − r2)^2/x. Cross belt: L = π(r1 + r2) + 2x + (r1 + r2)^2/x.
- Angle of contact is smaller on the smaller pulley; design for that pulley.

EXAMPLE Flat belt, lap angle 165°, μ = 0.3, T1 = 1500 N, v = 15 m/s (ignore Tc). Find power.

- θ = 165 x π/180 = 2.880 rad. e^(0.3 x 2.880) = e^0.864 = 2.372.
- T2 = 1500 / 2.372 = 632 N.
- P = (1500 − 632) x 15 = 13020 W ≈ 13.0 kW.

7. Flywheel and Turning Moment

- **Turning moment diagram**: torque on the crankshaft vs crank angle. Area = energy.
- **Flywheel** stores energy when supply > demand and gives it back when demand > supply. It controls speed changes within one cycle. (A governor controls the mean speed over many cycles.)

FORMULA Max fluctuation of energy ΔE = I ω^2 Cs = m k^2 ω^2 Cs. Coefficient of fluctuation of speed Cs = (ω1 − ω2)/ω.

EXAMPLE Engine at 300 rpm, total speed fluctuation 2% (Cs = 0.02), ΔE = 10 kJ. Find I.

- ω = 2π x 300 / 60 = 31.42 rad/s, ω^2 = 987.
- I = ΔE / (ω^2 Cs) = 10000 / (987 x 0.02) = 506.6 kg·m^2.

8. Governors

- Governor keeps the mean speed constant when load changes, by controlling fuel supply.
- Types: centrifugal (Watt, Porter, Proell, Hartnell, Hartung, Wilson–Hartnell) and inertia governors.

FORMULA Watt governor height h = g / ω^2 = 895 / N^2 m (N in rpm). Porter (equal arms): h = (m + M)/m x 895 / N^2.

- At 60 rpm, Watt governor h = 895 / 3600 = 0.249 m. Watt governor is used only at low speeds because h becomes too small at high speed.

Term	Meaning
Sensitiveness	Small change in speed causes a large sleeve movement
Stability	Radius of rotation increases as speed increases
Isochronous	Same equilibrium speed at all radii (infinitely sensitive)
Hunting	Governor too sensitive; speed keeps swinging above and below mean
Effort	Mean force on the sleeve for a given speed change
Power	Effort x sleeve lift

REMEMBER An isochronous governor is not practical because it hunts. Hartnell (spring-loaded) can be made isochronous.

9. Balancing and Gyroscope

➤ Balancing of rotating masses

- **Static balance**: $\Sigma m r = 0$ (vector sum). **Dynamic balance**: $\Sigma m r = 0$ and $\Sigma m r l = 0$ (couples about a reference plane).
- Masses in several planes can be balanced by two masses in two chosen planes.

➤ Balancing of reciprocating masses

FORMULA Primary force = $m \omega^2 r \cos\theta$. Secondary force = $m \omega^2 r \cos 2\theta / n$ ($n = l/r$).

- Primary force acts at crank speed; secondary at twice crank speed.
- Single-cylinder engines are only **partially balanced**: usually 2/3 to 3/4 of the reciprocating mass is balanced.
- Locomotives: partial balancing causes **variation in tractive effort**, **swaying couple** and **hammer blow**.

➤ Gyroscope

FORMULA Gyroscopic couple $C = I \omega \omega_p$ (ω = spin speed, ω_p = precession speed).

- Effects: ship steering, pitching and rolling; aeroplane turning; two- and four-wheelers on curves.

- Rule: spin vector, precession vector and couple vector follow the right-hand rule.

➤ Brakes and dynamometers (short)

- Block brake, band brake (T1/T2 = e^(μθ)), internal expanding shoe brake.
- Absorption dynamometers: prony brake, rope brake. Transmission: belt transmission, torsion dynamometer.

10. Most Asked Questions (practice)

1. Define kinematic pair, chain and mechanism. Classify kinematic pairs.
2. Find degrees of freedom of given mechanisms using Grubler's equation.
3. Explain inversions of single and double slider crank chains.
4. State and prove Kennedy's theorem. Locate ICs of a four-bar mechanism.
5. Explain Coriolis acceleration and when it appears.
6. Draw the cam profile for a roller follower with SHM / cycloidal motion.
7. State the law of gearing. Compare involute and cycloidal teeth. Explain interference.
8. Solve an epicyclic gear train by the tabular method.
9. Derive T1/T2 = e^(μθ) and the condition for max power in belts.
10. Explain governor sensitiveness, stability, isochronism and hunting.

TIP In GATE ME, focus on Grubler DOF, IC location, Coriolis, gear train ratios (epicyclic), flywheel ΔE, governors and gyroscopic couple. These give quick sure marks.